Topic 4.2.1 – Introduction to Filters

Learning Objectives:

At the end of this topic you will be able to;

- ☑ know and recall that the audio frequency range is approximately 20 Hz to 20 kHz;
- ☑ recall that high quality music transmission requires the full audio range.
- ☑ recall that the tonal quality of the received signal depends on the channel bandwidth allocated to it within the transmission system;
- ☑ recall that recognisable speech can be transmitted using a limited 300Hz to 3 kHz range to reduce the bandwidth requirement;
- ☑ understand that a complex wave is constructed from a fundamental frequency plus a number of harmonic frequencies;
- ☑ draw the frequency spectrum of a sine wave and a square wave (qualitatively) before and after passing through an ideal filter with a given frequency spectrum.

Module ET4 – Communication Systems

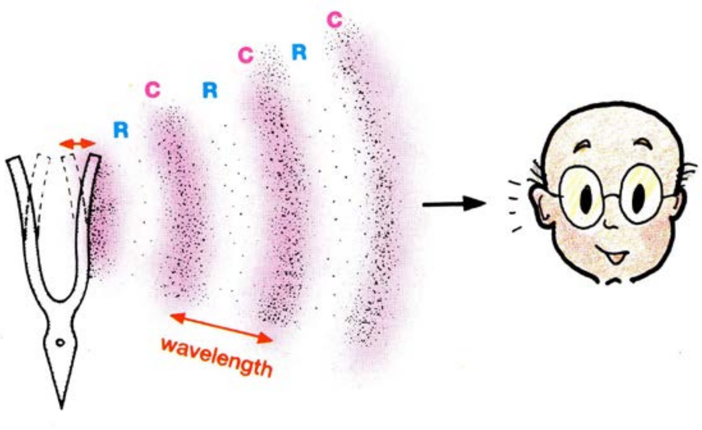
Introduction to Filters

The aim of this unit is to provide a basis for the work to follow on filters. It will introduce some new concepts as well as refresh your memories of work done in GCSE Science.

A filter as its name suggests is a device which allows only certain things through it just like filter paper in Chemistry can be used to separate mixtures of solid particles suspended in a liquid, provided that the holes in the filter are smaller the solid particles and big enough to let the liquid through.

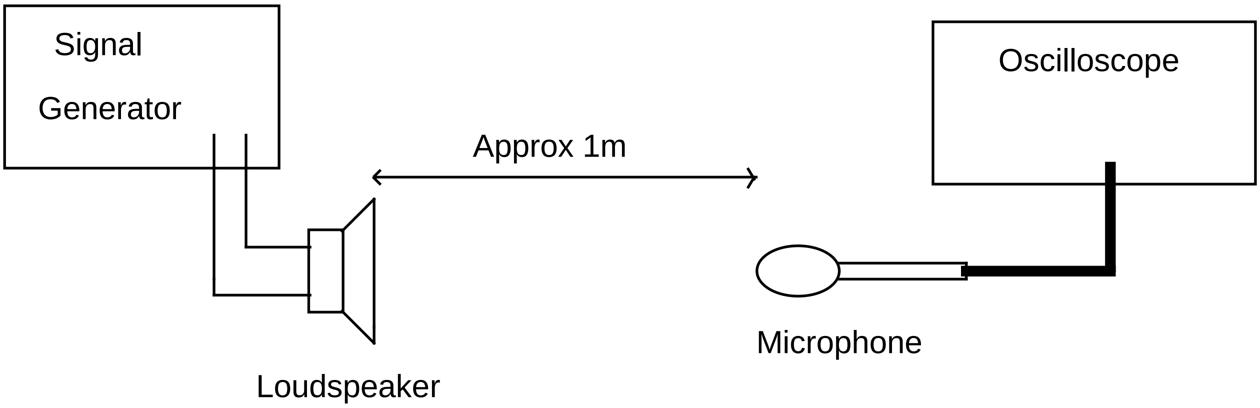
Filters used in electronics allow certain parts of an electronic signal to pass through while stopping other parts. In general filters are used in the communications field and you will have the opportunity of examining all of these later in the module.

To begin with we need to understand a little more about sound waves and the way in which these cause an electrical signal to 'wobble' or **oscillate.** From your work in GCSE Science you should recall that a sound wave is a longitudinal wave. It is created by something which is vibrating compressing and rarefying the air particles in front of it. This wave passes through the air until it strikes our ear drum, making that vibrate at the same rate as the air particles and this in turn causes part of the cochlea to vibrate and we hear the sound. This is shown in the diagram below.

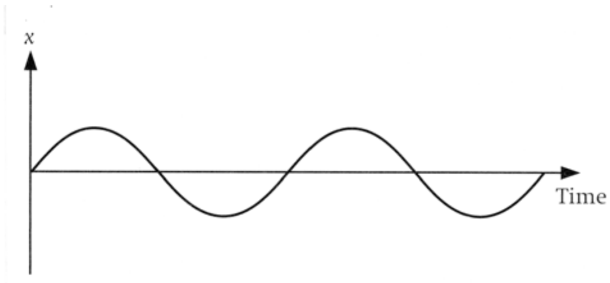


Demonstration:

We can recreate this process using a signal generator, loudspeaker, microphone and an oscilloscope, set up as follows.



The signal generator should be set to produce a **1 kHz sine wave,** a wave that has the following shape.



This is applied to the loudspeaker, and a sound will be produced which is constant in pitch. The sound is made by the paper cone of the loudspeaker moving in and out (if you touch the cone carefully with your finger, you will feel the vibrations), causing the air in front of it to be compressed or rarefied. The speed at which the paper cone moves in the loudspeaker is controlled by the number of waves that arrive at the speaker each second. This is called the **frequency** of the wave. 1 kHz means that 1000 waves are being produced each second. If you set the signal generator to a low frequency (1 to 2 Hz) you will be able to see the cone of the loudspeaker moving in and out.

Using a microphone to act as our ear, we can pick up the sound created by the loudspeaker and display this on an oscilloscope. The microphone contains a thin material, just like our eardrum, which vibrates when sound waves strike it. This causes tiny electrical signals to be generated, which can be displayed on the oscilloscope. The display on the oscilloscope should be a sine wave of the same frequency as that generated by the signal generator.

This setup can be used to determine your range of hearing by slowly decreasing the frequency of the signal generator from 200 Hz until you can no longer hear the low pitched sound. This will be the lowest frequency you can hear. Compare this to the lowest frequency that can be detected by the microphone on the oscilloscope.

Lowest Frequency to be heard = ………………………………….

Lowest Frequency displayed on the oscilloscope = …………………………………….

A similar investigation can be carried out at the high frequency end. Set the signal generator to 10 kHz and slowly increase the frequency until you can no longer hear the high pitched sound from the loudspeaker. This will be the upper limit of your hearing. Check to see what frequency the microphone stops working at.

Highest Frequency to be heard = ………………………………….

Highest Frequency displayed on the oscilloscope = …………………………………….

If you compare the results of the class, there is likely to be some variation in the range of frequencies you can hear, especially if you include your teacher. This is because as people get older their range of hearing decreases, particularly at the high frequency end.

The range of human hearing is identified as being within the range 20Hz to 20,000 Hz or 20 kHz. How did your range compare to this ?

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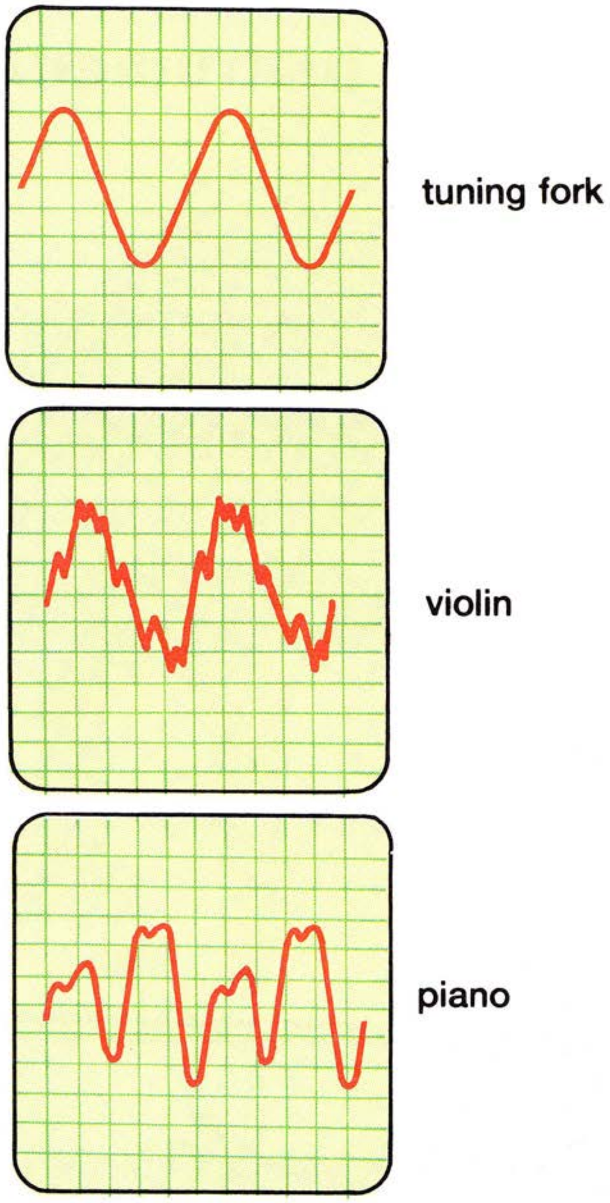
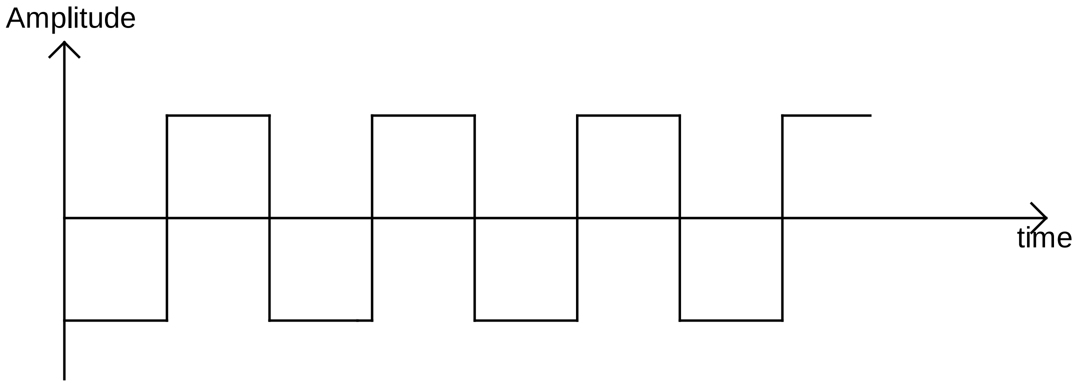
So far we have only considered one type of wave, the sine wave, in our discussion of sound. The following pictures show the trace produced on an oscilloscope of various musical instruments playing the same note.



It does not take long to realise that even though the same note is being played, the sound waves produced look completely different. This is because there is another quality of sound that we have not mentioned yet and that is **tone.** To understand this we need to look at two very different but simple looking waves, the sine wave and the square wave.

We are already familiar with the sine wave, we have just used it to determine our range of hearing. A square wave on the other hand looks very simple and can also be generated by the signal generator.



Set the signal generator to produce a 1 kHz sine wave, then switch to a square wave, can you tell the difference ? Complete the following table. (If you are not carrying out the demonstration ask your teacher for the results)

Frequency (kHz)	Is there a difference in what you hear between sine wave and square wave ?
1	
2	
3	
4	
5	
6	
7	
8	

Completing the table should have revealed a strange result, at around 6kHz it becomes impossible to determine the difference between a sine wave and a square wave, but why is this, we can see both waves on the oscilloscope and they look completely different, why do they sound the same ?

The answer lies in the fact that a square wave is not really a square wave at all, it is made up from an infinite number of sine waves, all with different frequencies and amplitudes. This fact was discovered by Jean Baptiste Fourier, a mathematician, who explained that a waveform no matter how complex can be made up of a series of sine waves having different amplitudes and frequencies; this is called **Fourier Analysis,** and there is a whole branch of mathematics devoted to understanding and applying his theories.

Fortunately we do have to go into this in too much depth for this course, what we need is to be able to understand the consequences of his work, and its role in communications. Fourier's work basically concluded that any waveform starts with the **fundamental**, or **first harmonic**, which has the same frequency, f, as the original waveform. The series then continues with the **second harmonic,** of frequency, $2f$, the **third harmonic,** of frequency, $3f$, and so on.

The amplitudes of the various harmonics depend on the shape of the original waveform. The simplest waveform will only have a fundamental and no other harmonics; this is a pure sine wave. Such a wave in sound is often referred to as a **pure tone**.

A square wave on the other hand is anything but simple. Fourier Analysis of a square wave with **equal** mark / space ratio reveals that it is made up from a fundamental at the frequency of the square wave added to an infinite number of odd harmonics of ever decreasing amplitude. The expression for such a wave is:

7th Harmonic

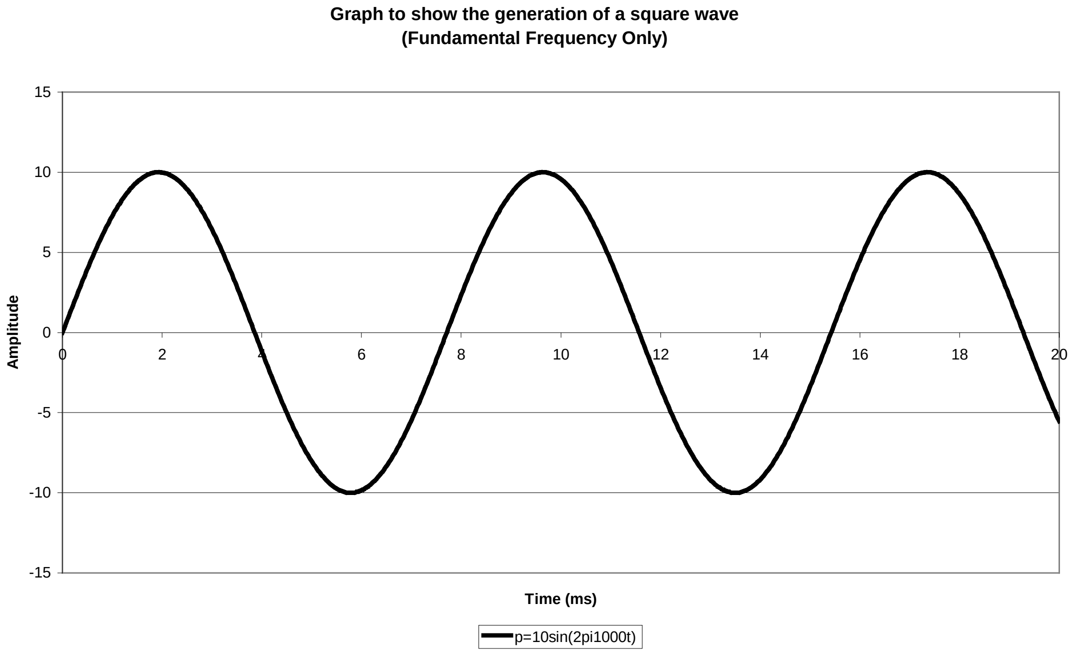
$$y = A\sin 2\pi ft + \frac{A}{3}\sin 2\pi 3\,ft + \frac{A}{5}\sin 2\pi 5\,ft + \frac{A}{7}\sin 2\pi 7\,ft + ...etc$$

Fundamental | 3rd Harmonic | 5th Harmonic

where y = displacement at time t, A is the amplitude of square wave, and f is the frequency.

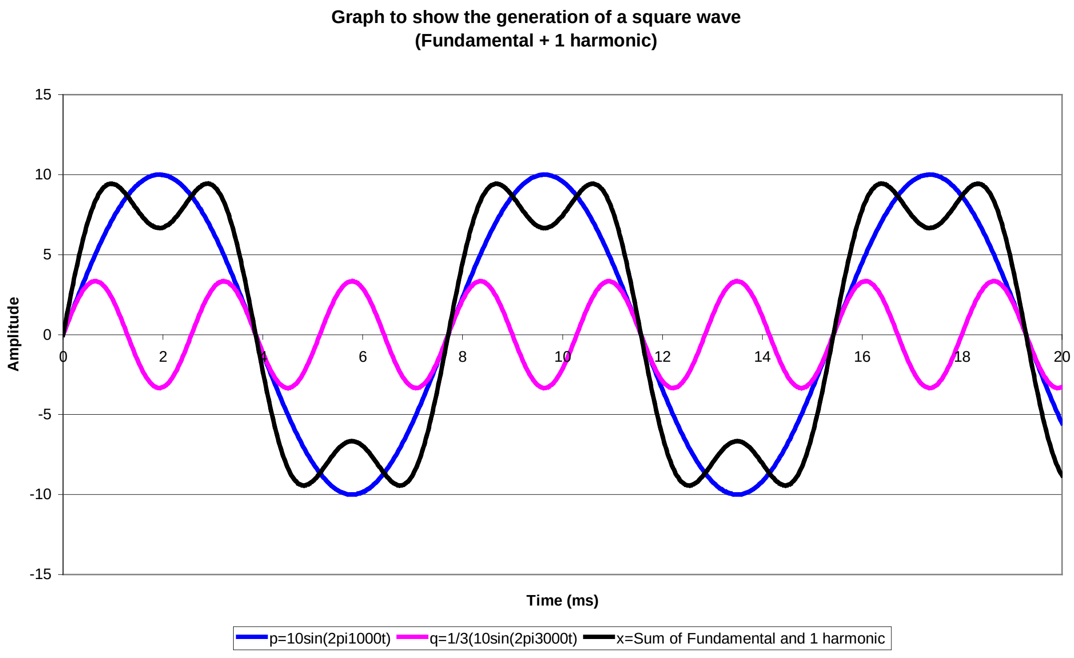
The following diagrams illustrate what happens as we keep adding these harmonics:

Firstly – just the fundamental.



Next we will add the third harmonic which is given by the equation

$$y = \frac{A}{3}\sin 2\pi 3ft$$



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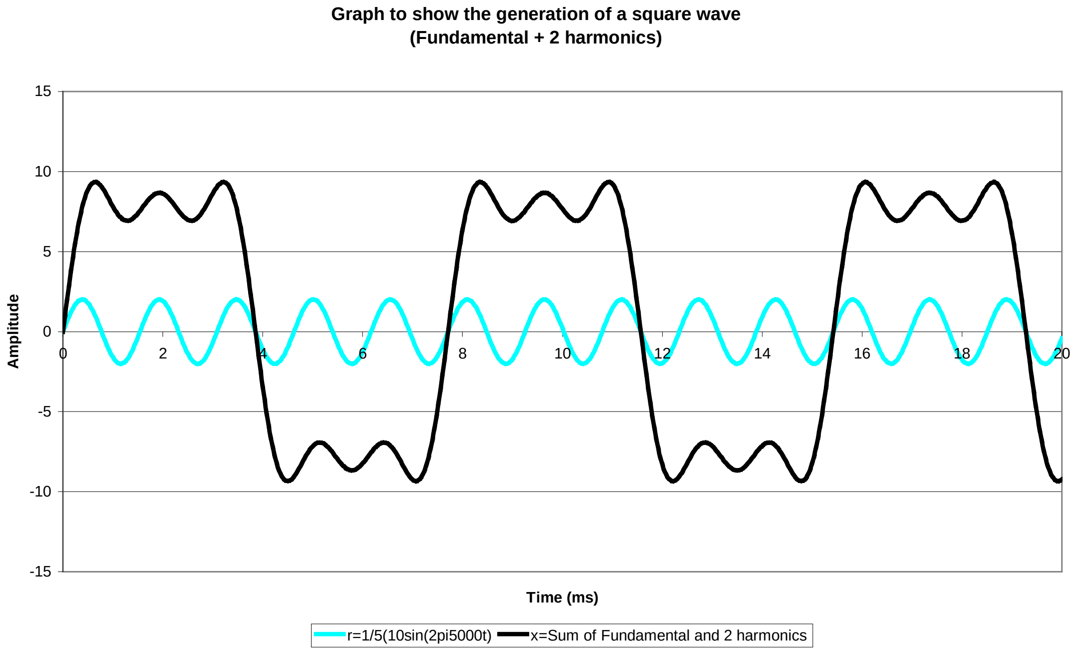
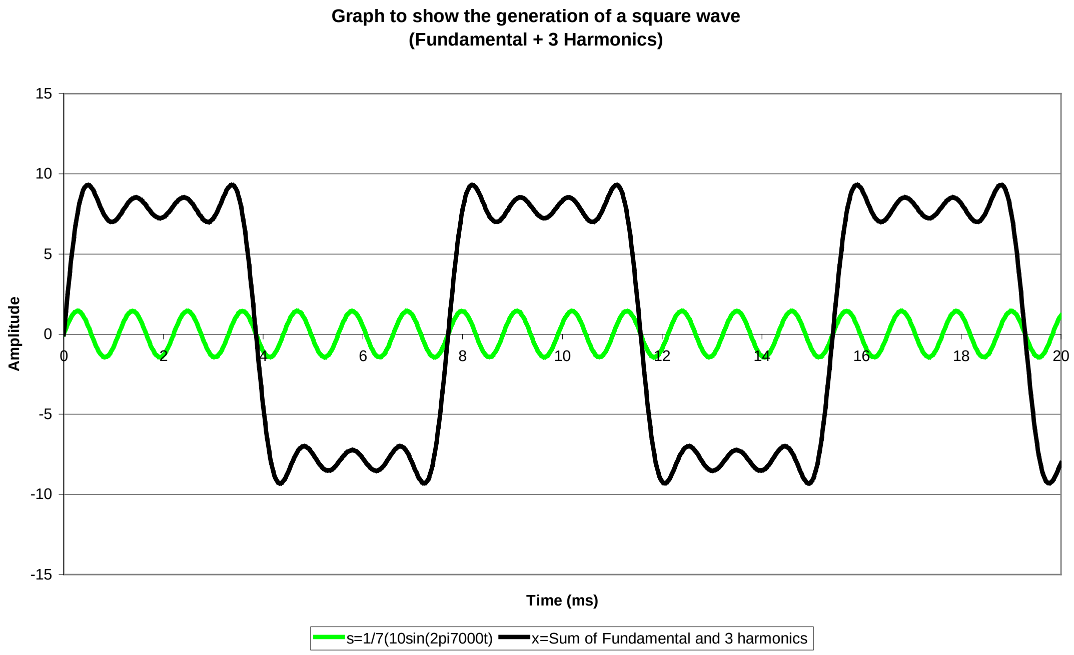
Now we will add the fifth harmonic which is given by the equation

$$y = \frac{A}{5}\sin 2\pi 5\,ft$$



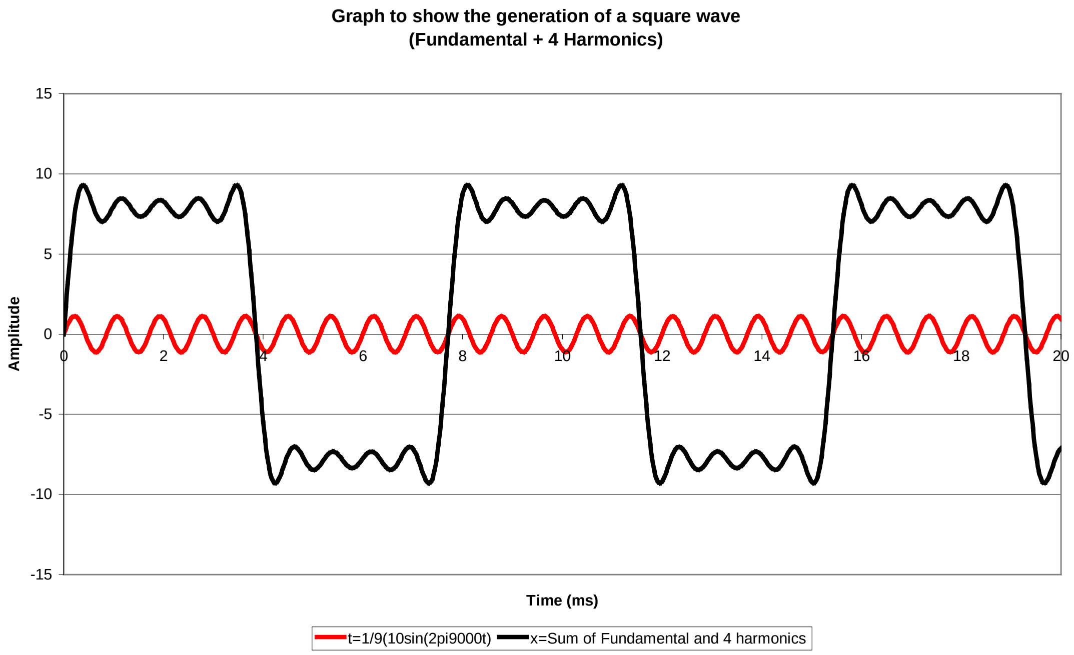
Now we will add the seventh harmonic which is given by the equation

$$y = \frac{A}{7}\sin 2\pi 7\,ft$$



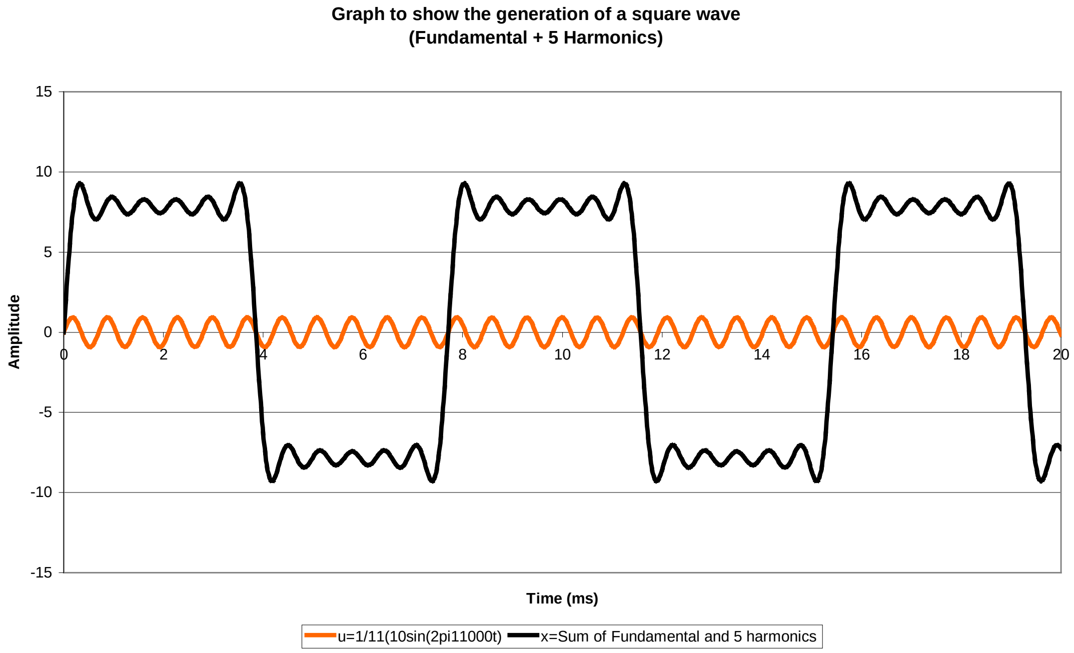
Now we will add the ninth harmonic which is given by the equation

$$y = \frac{A}{9}\sin 2\pi 9\,ft$$



Finally we will add the eleventh harmonic which is given by the equation

$$y = \frac{A}{11}\sin 2\pi 11\,ft$$



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Hopefully you are now in a position to see exactly what is happening every time we add an extra harmonic to the graph. The edges of the graph are becoming more vertical and sharper, and the crest and trough are getting flatter.

We have shown the addition of 5 harmonics here and we are still a long way off the 'perfect' square wave. Remember that Fourier analysis has shown that to create the perfect square wave we need an infinite number of odd harmonics, which is clearly impractical to do here, but you will hopefully understand the principle.

So let's return now to our original problem – why does a 6 kHz sine wave and square wave sound the same. Well if we think of how the square wave is formed it is made up of sine waves equal to the fundamental frequency and an infinite number of 'odd' harmonics, so for a 6KHz square wave this means frequencies of 6 kHz (the fundamental), then harmonics at frequencies of 18kHz (3 x 6 kHz), 30 kHz (5 x 6 kHz), 42 kHz (7 x 6 kHz) etc.

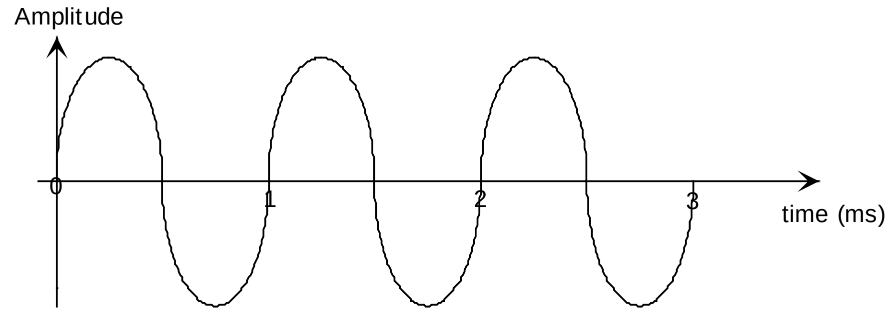
Now if we think back to the human range of hearing which was just 20 Hz to 20 kHz, the first harmonic at 18 kHz is already approaching the limit of what humans can hear, a few people might be able to distinguish just a slight variation between the two. The majority of the population however would just hear the fundamental, which is a sine wave of frequency 6 kHz, so it is not surprising that it sounds the same as a 6 kHz sine wave, because they are indeed identical!

By the time you reach a frequency of 7 kHz there is no discernable difference for anyone as the first harmonic at this frequency is at 21 kHz (3 x 7 kHz) which is already outside of the range of human hearing.

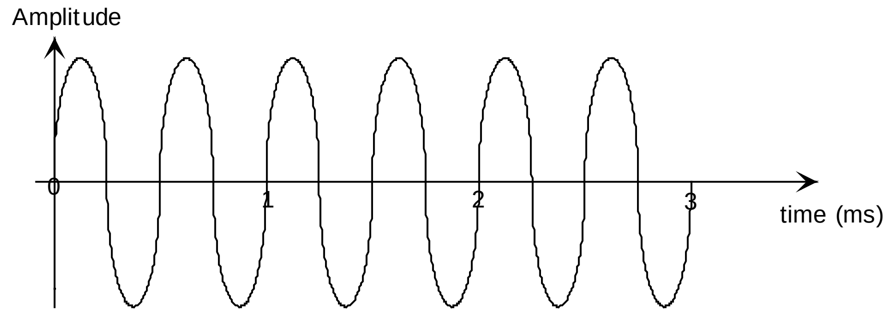
Frequency Spectrum.

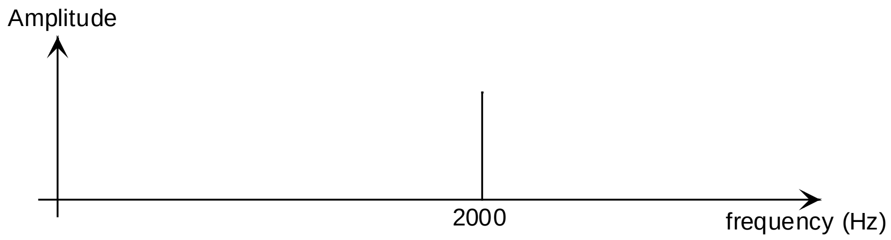
We are used to drawing graphs of wave forms with 'time' being the unit of measure on the x-axis. We can also draw a graph with frequency on the x-axis instead of time. This graph is known as the **spectrum** of the waveform.

For a pure sine wave of frequency 1 kHz the two ways in which we can represent the signal are as follows:

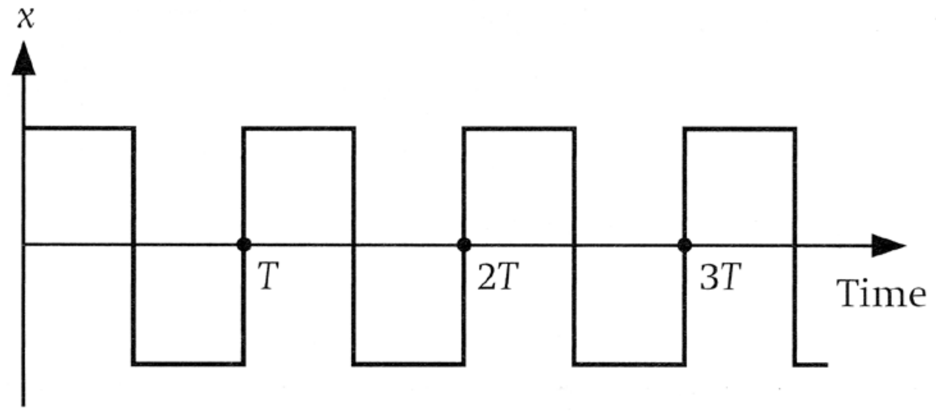


For a 2 kHz Sine wave these will become:



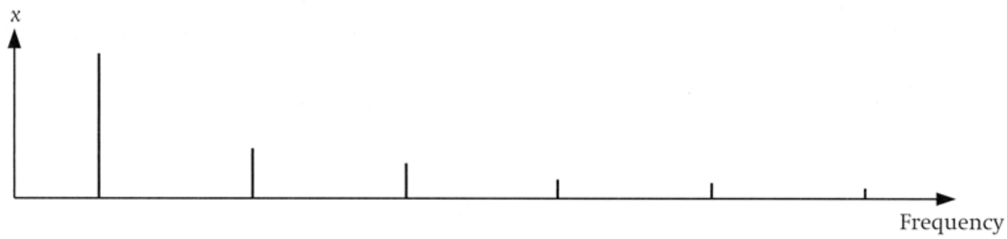


If we were to draw a square wave having a periodic time *'T'* then the graph below would result.



We know from our previous work that in practice this square wave is made up of a number of sine waves at increasing frequency, and decreasing amplitude.

We have seen that the spectrum for a pure sine wave is simply a straight line (i.e. it is composed of only one frequency). When we examined the structure of a square wave we found that it is made of an infinite number of sine waves with ever decreasing amplitude. This results in a frequency spectrum as shown below:



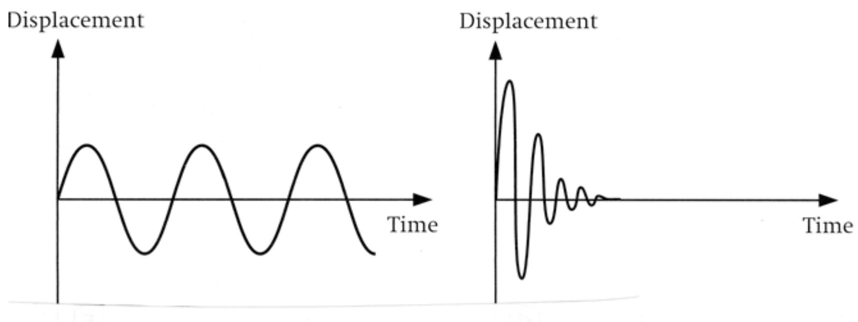
In telecommunications all signals have waveforms that can be analysed in this way to determine their frequency content. By analysing the frequency content we are able to determine the **Bandwidth** needed to transmit the signal. The Bandwidth is the range of frequencies required to make up the signal. i.e. the bandwidth equals the highest frequency component minus the lowest frequency component.

This means that for a perfect sine wave, the bandwidth is zero, since there is only one frequency present in the original signal. However the bandwidth of a perfect square wave is infinite, because to construct a perfect square wave we require an infinite range of sine waves. Every other signal we can think of will lie between these two extremes. Sometimes engineering decisions are taken to limit the bandwidth of certain signals because the effect at the receiving end is either not noticeable or an acceptable quality of communication is achieved for that particular purpose.

Let us have a look at the bandwidths required for typical audio and video information.

Audio Information – Speech.

We speak by making the vocal chords in our throat vibrate. This moves the surrounding air molecules in such a way that they become compressed or rarefied and this creates a longitudinal wave in the air. The sound we make is a complex wave as we produce different sounds. The following graphs show the sound waves produced for a vowel and a consonant.



A typical voice produces a power of about 15μW, but it is not evenly distributed over the audio frequency range. When the human voice is analysed for its frequency content, i.e. a frequency spectrum, the following are obtained:

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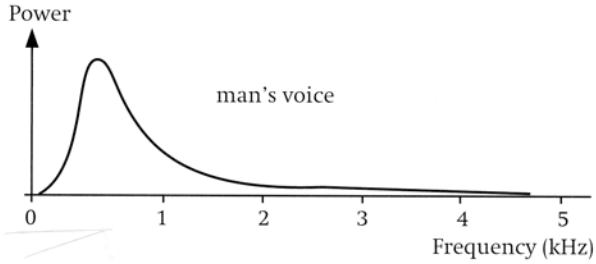
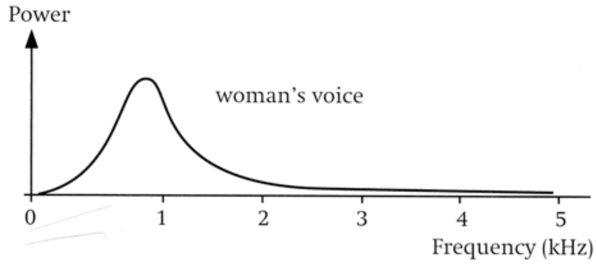




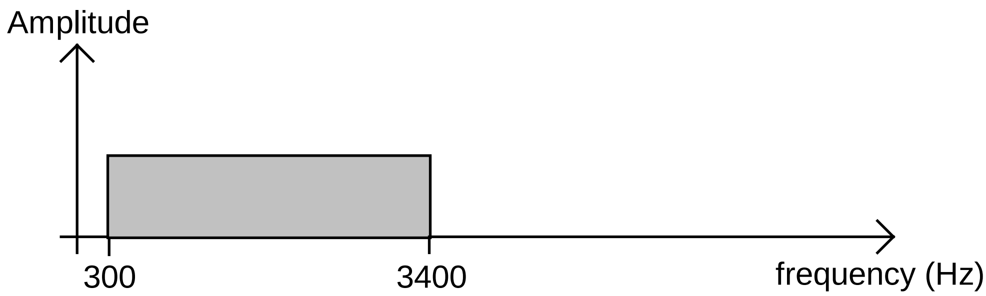
Note:
i. that the spectra produced by the human voice shows not single frequencies but a whole spread giving rise to the shape of the graphs shown.
ii. the peak of the frequency graphs is different for a male and female.

By looking at the spectra more closely, you may be surprised to find that most of the power of the voice lies in the lower frequency end of the spectrum, as this is where the peak is located. This does not mean that we do not produce higher frequencies than this, indeed we can produce a range from about 100 Hz up to 10 kHz, but the power at these frequencies is very small and has a minimal effect on the actual sound.

Telecommunications companies have utilised this spectrum to decide on how much bandwidth they are going to allocate to each user in their system. If you look carefully at the spectra once more you can see that there is very little power produced after 2 kHz, and you might suspect that this might be the chosen range, however, during trials it was found that using 2 kHz as the bandwidth there was some confusion over the 's' and 't' sounds and so the range had to be made a little bit larger to accommodate more of the harmonic frequencies to enable these sounds to be distinguished clearly.

The public telephone system uses a bandwidth of 3.1kHz, with a frequency range of 300Hz to 3.4kHz. This range has proved to be acceptable in identifying a caller by being able to recognise their voice and understand simple conversations.

During normal conversation the human voice produces a constantly changing range of amplitude and frequency, particularly when local accents are taken into account. Therefore we are not certain exactly what frequencies and amplitudes are present in a particular conversation – all we know is that there is a maximum amplitude plus a maximum bandwidth which is usually represented by a rectangle of frequencies. If we use a telephone call as an example, the range of frequencies will be between 300 Hz and 3.4 kHz, so the spectrum will look like this.



Audio Information – Music.

Our ears are much more sensitive to a greater range of frequencies than those transmitted by a normal telephone. If you have ever listened to music being played down a telephone you will know that it sounds very poor.

In general, music produced from a variety of instruments will cover a range from about 20 Hz to about 20 kHz. This is significantly above the 3.1kHz bandwidth limit of the telephone so it is no wonder the music is of poor quality.

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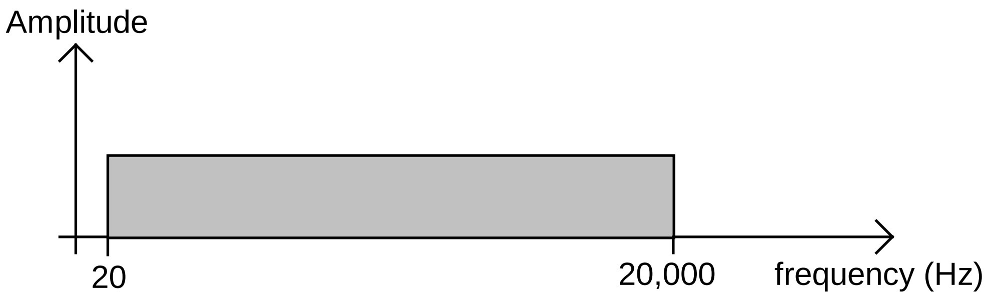
In practice to keep costs down, broadcasters of music restrict the range actually transmitted to suit the bandwidth of the channel being used for broadcasting. The BBC has two standards **Hi-fidelity** (20Hz to 15kHz) used on the very high frequency (VHF) radio, and **Low-fidelity** (100Hz to 3.5kHz) used on long-wave (LW) and medium-wave (MW) radio.

We represent the frequency spectrum of a music signal in a similar way to that of a speech signal, it is just that the band is wider as shown below.



Ideal Filters

Now that we understand how a complex waveform is made up of a number of different frequencies, and we can draw the spectrum of a sine wave and a square wave, we are in a position to look at the role of filters in communication systems.

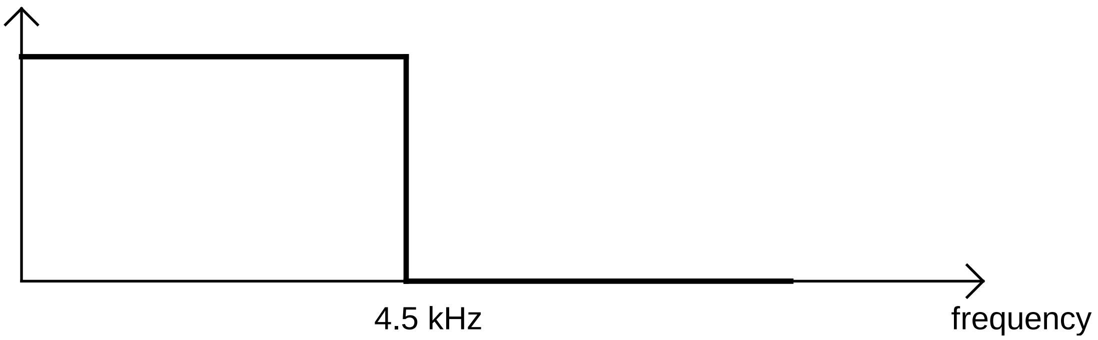
Filters fall into three main categories:

i. Low Pass Filter (LPF)
ii. High Pass Filter (HPF)
iii. Band Pass Filter (BPF)

In the next section we will look at how these filters are made from components, but for now we will just consider their basic function and operation and pretend that they are perfect or 'ideal' in what they do.

i. The Low Pass Filter.

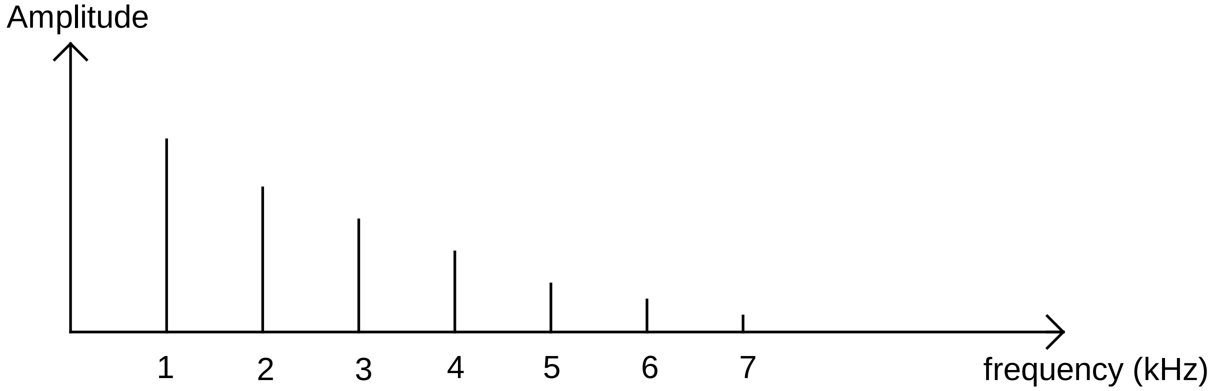
As its name suggests this type of filter allows low frequency signals to pass through unaffected, but high frequency signals are blocked. It can be represented by a frequency spectrum like this:



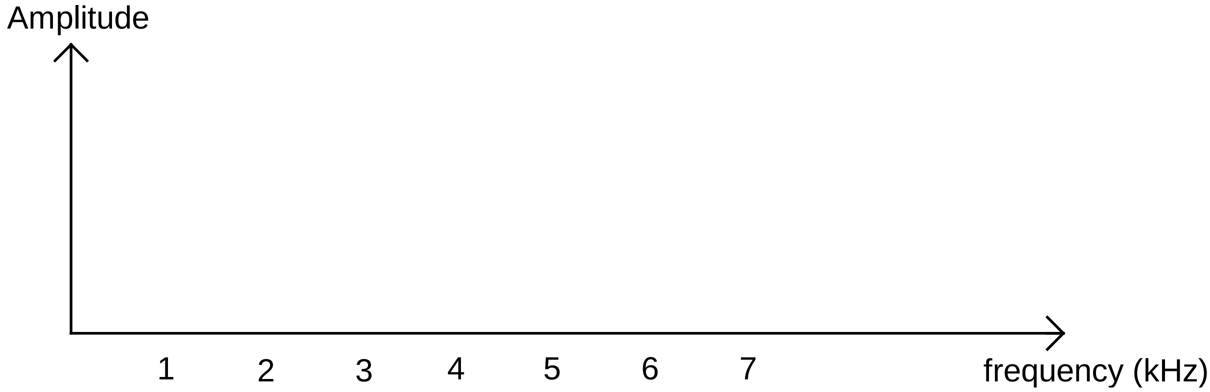
For the filter represented by the characteristic above all frequencies below 4.5 kHz would be allowed through without any changes. All frequencies above 4.5 kHz would be blocked and no trace of them would appear at the output.

If a complex signal having the following frequency spectrum was applied to the LPF above, what would the output spectrum look like?

Input Spectrum:

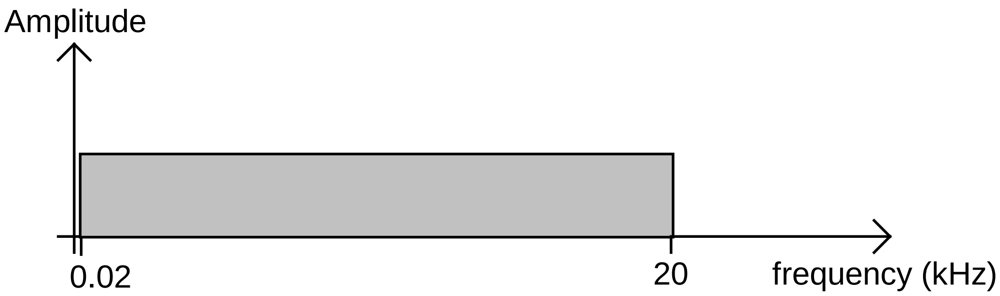


Output Spectrum:

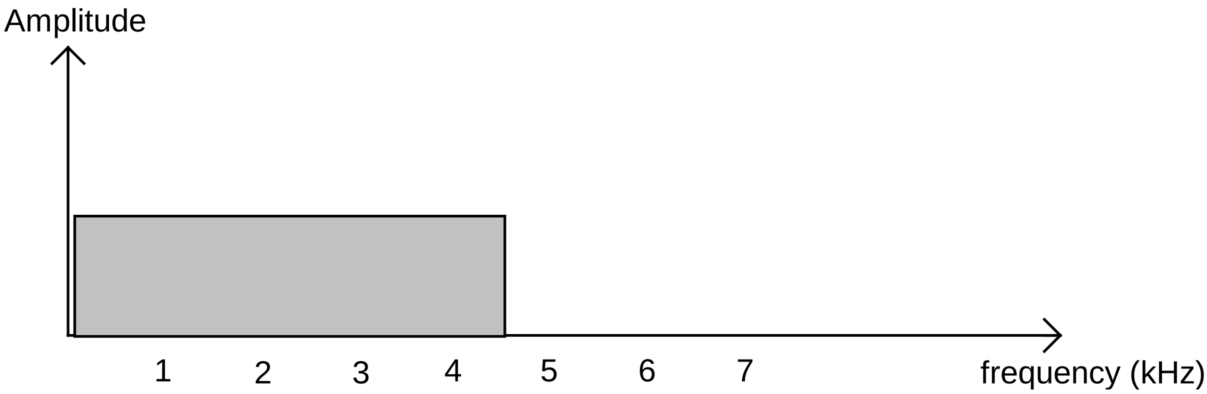


If a music signal, having the following spectrum were applied to the low pass filter, what would the output look like ?

Input Signal.



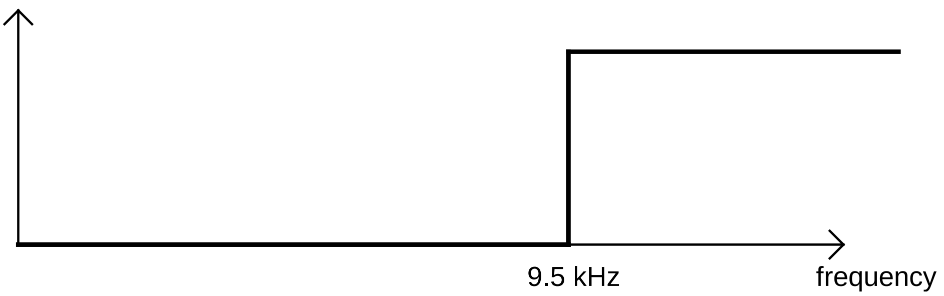
Output Signal:



Note : that even though the bandwidth has been severely cut back, the signal will still be distinguishable. It is the quality that has been lost by cutting off the high frequency components.

ii. The High Pass Filter.

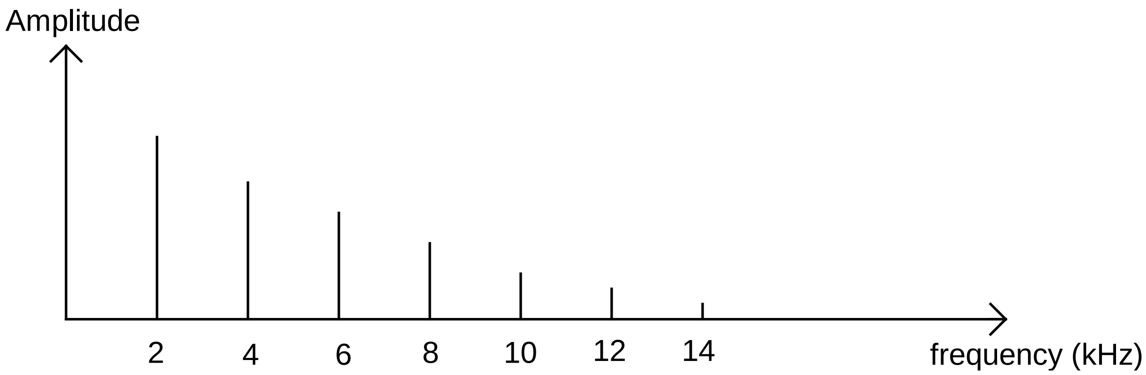
As its name suggests this type of filter allows high frequency signals to pass through unaffected, but low frequency signals are blocked. It is the opposite to a Low Pass Filter. It can be represented by a frequency spectrum like this:



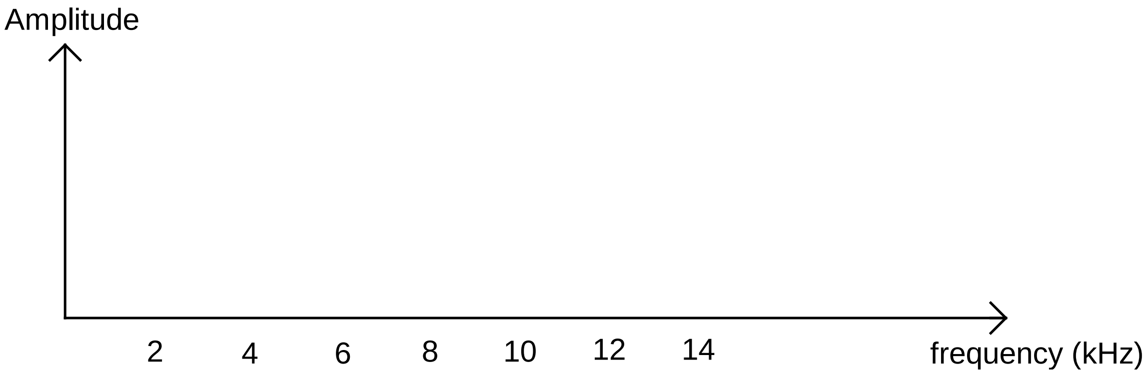
For the filter represented by the characteristic above all frequencies above 9.5 kHz would be allowed through without any changes. All frequencies below 9.5 kHz would be blocked and no trace of them would appear at the output.

If a complex signal having the following frequency spectrum was applied to the HPF above, what would the output spectrum look like?

Input Spectrum:

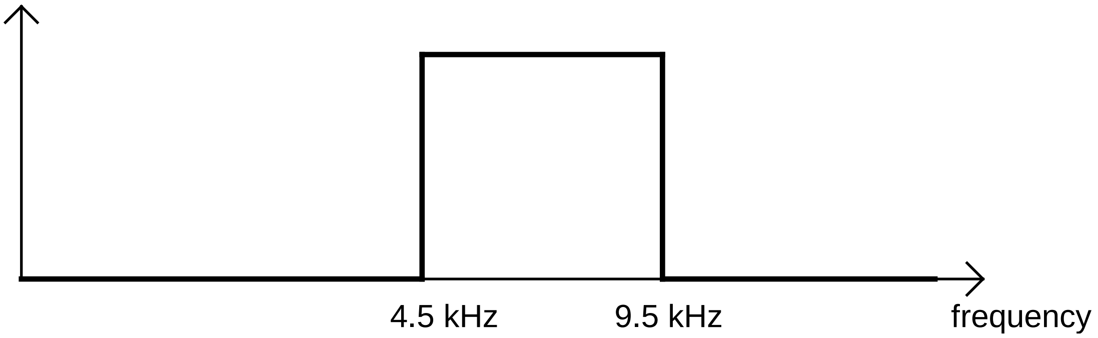


Output Spectrum:



iii. The Band Pass Filter.

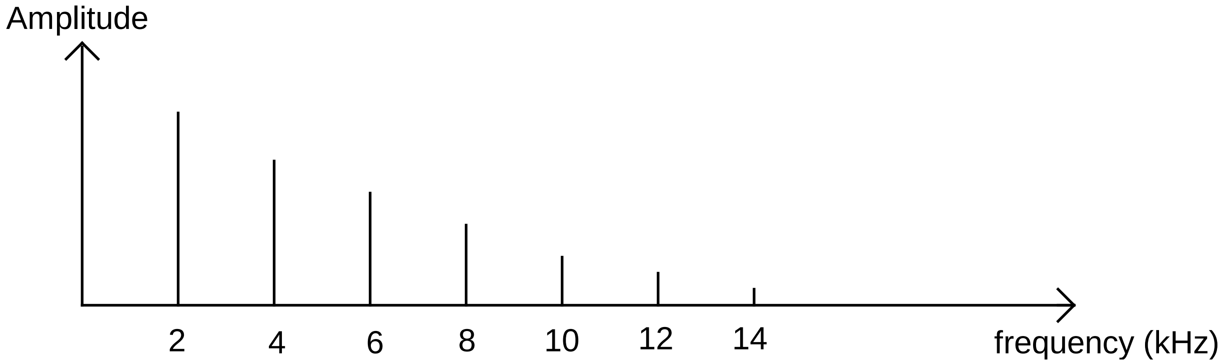
This is a special type of filter that only allows a certain range or 'band' of frequencies to pass through unaffected, but any signal having a frequency outside of this range either lower or higher are blocked. It can be represented by a frequency spectrum like this:



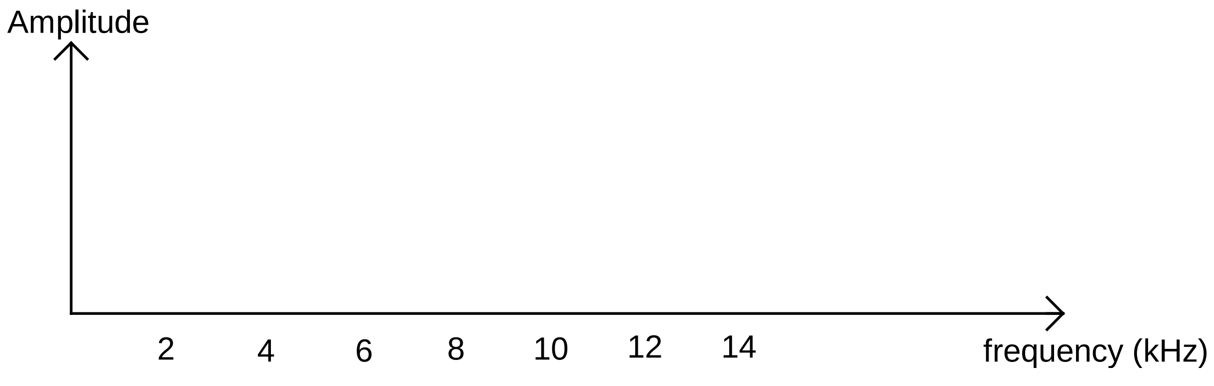
For the filter represented by the characteristic above all frequencies above 4.5 kHz and below 9.5 kHz would be allowed through without any changes. All frequencies below 4.5 kHz and above 9.5 kHz would be blocked and no trace of them would appear at the output.

If a complex signal having the following frequency spectrum was applied to the BPF above, what would the output spectrum look like?

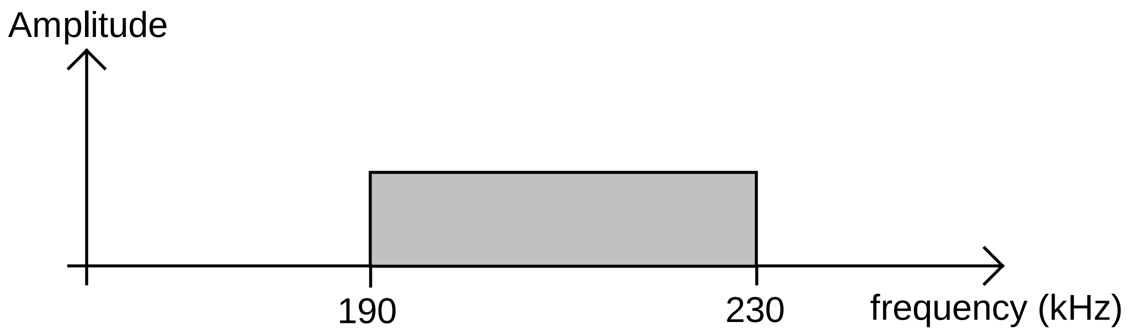
Input Spectrum:



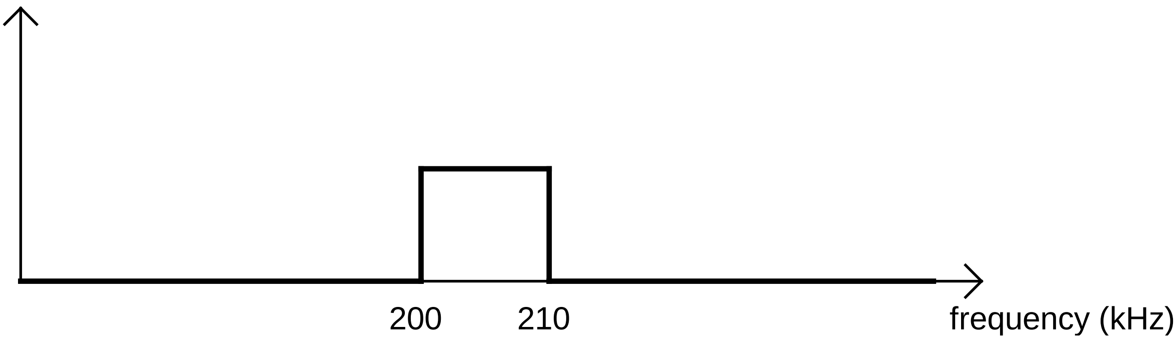
Output Spectrum:



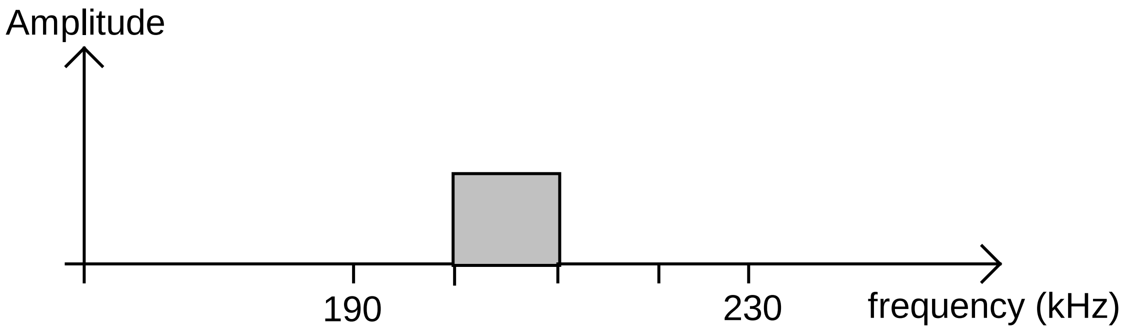
A radio signal has the following spectrum.



This radio signal is applied to a band pass filter with the following characteristic.



The output of the filter will be as follows:

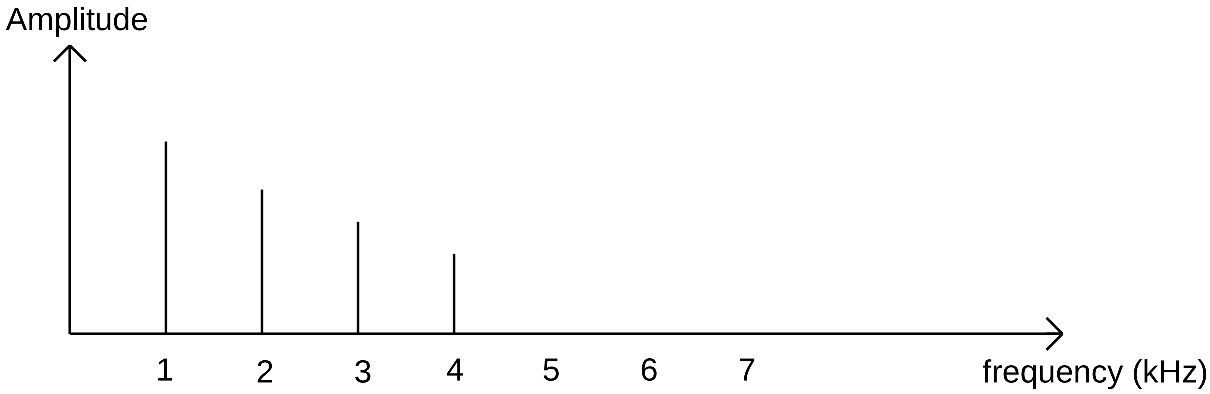


What we have considered here is the ideal characteristics of a filter, i.e. what we would really like it to do. As is usually the case when we try to build such filters we cannot quite obtain the ideal characteristic we are aiming for but we do get very close. In the next section you will learn how to construct, and design these filters for a variety of applications, including how to set the frequency at which they operate.

Solutions to in text questions.

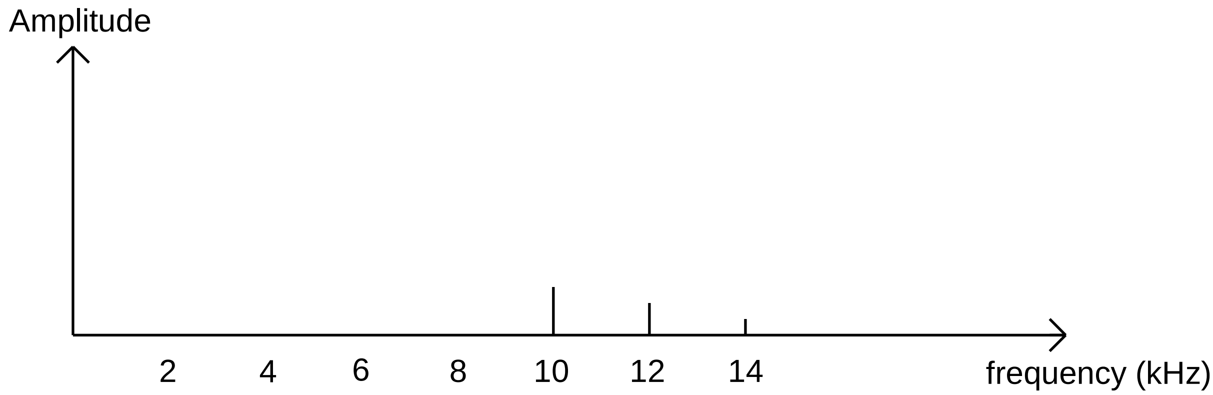
i. Low pass filter.

Output Spectrum.



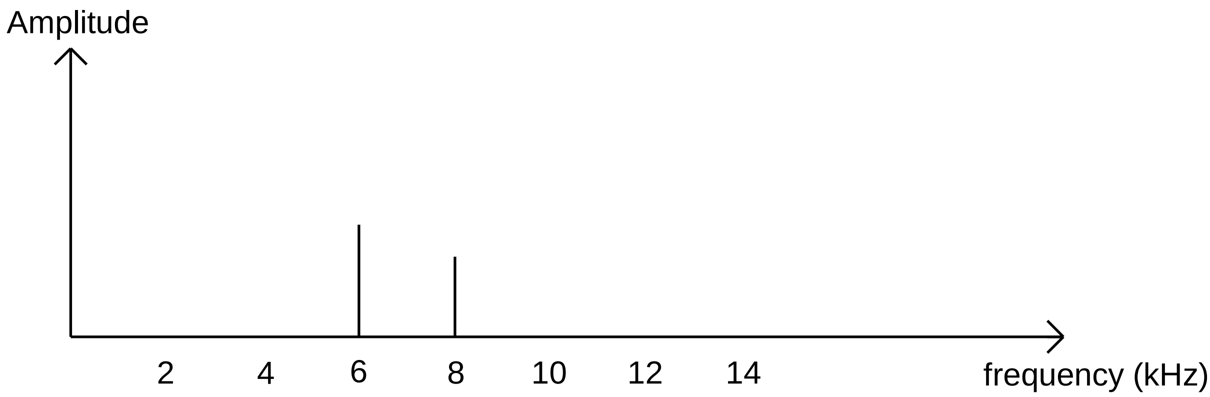
ii. High pass filter.

Output Spectrum.



iii. Band pass filter.

Output Spectrum.



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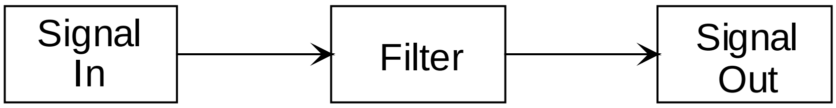
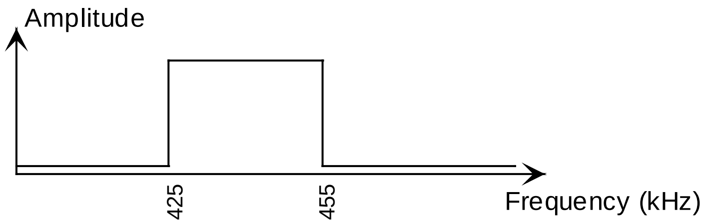
Module ET4 – Communication Systems

Practice Exam Question:

1. When studying the behaviour of filters and their use in electronic circuits a number of pupils were given various filter circuits to investigate.

One pupil was given the task of investigating the behaviour of a filter with the characteristic shown below, in response to different input signals. A block diagram of the test system arrangement is also shown below.

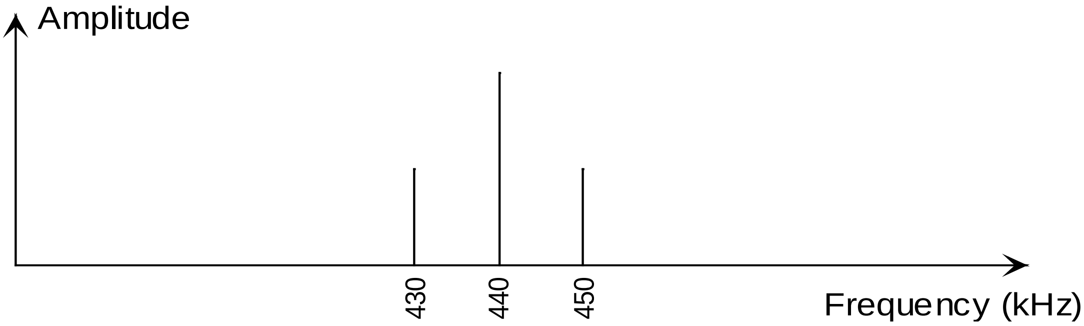




(a) What is the name of this type of filter ?

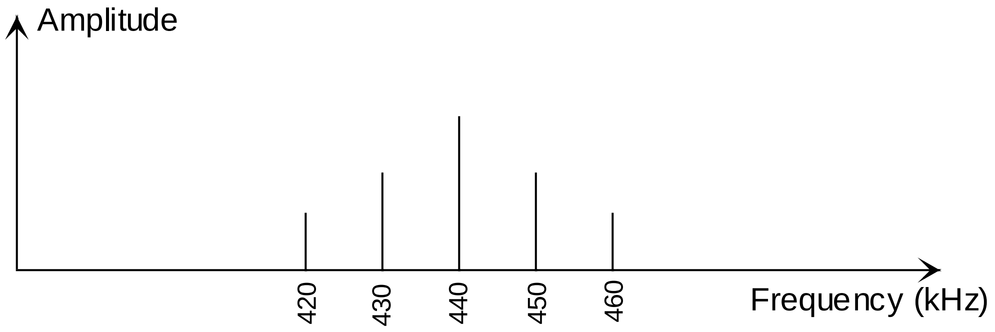
……………………………………………………………………………………………....... [1]

(b) The pupil then feeds a radio signal having the following frequency spectrum into the filter.



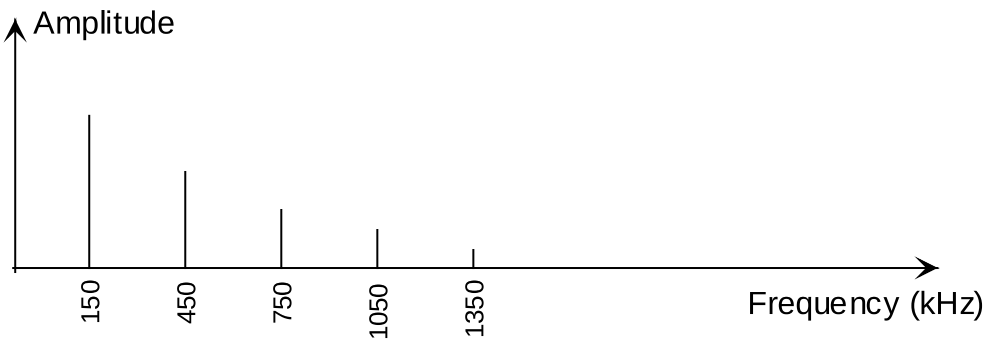
Sketch the frequency spectrum of the output you would expect the pupil to see at the output of the filter, labelling any relevant frequencies. [1]

(c) The pupil then feeds a different radio signal having the following frequency spectrum into the filter.



Sketch the frequency spectrum of the output you would expect the pupil to see at the output of the filter, labelling any relevant frequencies. [1]

(d) The pupil finally feeds a square wave signal having the following frequency spectrum into the filter.



Sketch the frequency spectrum of the output you would expect the pupil to see at the output of the filter, labelling any relevant frequencies. [1]

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Module ET4 – Communication Systems

Self Evaluation Review

Learning Objectives	My personal review of these objectives:		
	☺	😐	☹
know and recall that the audio frequency range is approximately 20 Hz to 20 kHz;			
recall that high quality music transmission requires the full audio range.			
recall that the tonal quality of the received signal depends on the channel bandwidth allocated to it within the transmission system;			
recall that recognisable speech can be transmitted using a limited 300Hz to 3 kHz range to reduce the bandwidth requirement;			
understand that a complex wave is constructed from a fundamental frequency plus a number of harmonic frequencies;			
draw the frequency spectrum of a sine wave and a square wave (qualitatively) before and after passing through an ideal filter with a given frequency spectrum.			

Targets: 1. ……………………………………………………………………………………………

……………………………………………………………………………………………

2. ……………………………………………………………………………………………

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