

Learning Objectives:

At the end of this topic you will be able to;

- ☑ recognise, analyse, and sketch characteristics for a low pass and a high pass filter;
- ☑ design circuits to act as low pass or high pass filters;
- ☑ select and use the formula $X_C = \frac{1}{2\pi fC}$;
- ☑ understand the significance of the term impedance, and that it is a function of X_C , and R in an R-C circuit;
- ☑ select and use the formula $Z = \sqrt{R^2 + X_C^2}$ to calculate the impedance of a series R-C circuit.
- ☑ define and calculate the break frequency, selecting and using the formula $f_b = \frac{1}{2\pi RC}$;
- ☑ plot and interpret graphs showing the frequency response of an R-C filter.

Filters

Filters fall into two main categories;

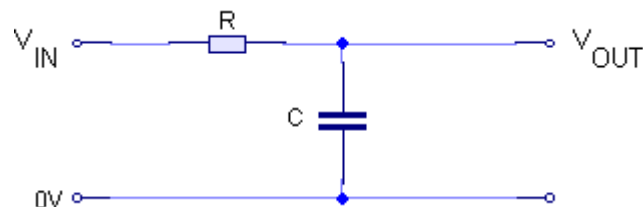
1. Passive Filters
2. Active Filters

In this section we are only going to investigate passive filters, active filters will be covered in Module ET5.

Passive filters can be used to **suppress** frequencies within a frequency spectrum. They can be made from combinations of resistors, capacitors and inductors.

i) A Simple Low Pass Filter.

A low pass filter (LPF) is used to remove high frequency signals from a signal spectrum. The circuit is very straightforward.



The circuit consists of a resistor in series with a capacitor. The output voltage is taken across the capacitor as shown.

In order to understand the way in which the circuit works we must remember that we are essentially dealing with an a.c. circuit. In an a.c. circuit capacitors behave in a different way to when they are in a d.c. circuit. We say that capacitors do not have resistance but **reactance**, to identify that it is in an a.c. circuit and it is given the symbol X_C . It is measured in Ohms (Ω)

Topic 4.2.2 - Passive R-C Filters



To calculate the reactance of the capacitor at any given frequency we can simply use the following equation.

$$X_c = \frac{1}{2\pi fC}$$

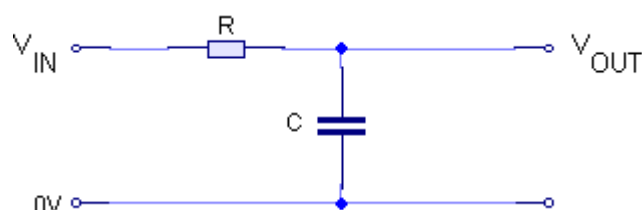
Where X_c is the reactance (measured in ohms), f is the frequency of the a.c. signal measured in Hertz and C is the value of the capacitance in farads.

Resistance is not affected by an a.c. signal and we do not have to use any different formulae to calculate resistance in an a.c. circuit.

When considering the effect of the resistance and capacitor in a circuit together, we define a new term for the combined effect of resistance and reactance as **impedance**, given the symbol Z . To find the total **impedance** in the circuit, we cannot simply add the reactance of the capacitor, to that of the resistor, again another formula is required. The total impedance of the circuit is measured in Ohms (Ω). The formula required is as follows:

$$Z = \sqrt{R^2 + X_c^2}$$

We will now go back then to the circuit and examine how we analyse the circuit and concentrate on this aspect rather than where the equations come from. The circuit is reproduced below for convenience.



If we consider the circuit to be a potential divider, albeit with an a.c. power supply, we can write down a formula for the output voltage in a similar way to how we would for a circuit containing two resistors. i.e.

$$\begin{aligned} V_{OUT} &= \frac{V_{IN}}{Z} \times X_c \\ &= \frac{V_{IN}}{\sqrt{R^2 + X_c^2}} \times X_c \end{aligned}$$

This can be re-arranged to look at the '**gain**' of the circuit. From your work in ET1 on op-amps you will remember that gain is defined as $\frac{V_{OUT}}{V_{IN}}$. It is a simple matter to obtain a formula for this from the equation above. i.e.

$$\frac{V_{OUT}}{V_{IN}} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$

Taking a look at the two extremes, i.e. very low and very high frequencies.

When the frequency is low, $X_C^2 \gg R^2$, R^2 will be so small it can be ignored compared to the size of X_C^2 and the gain will be near 1. i.e. no change.

Topic 4.2.2 - Passive R-C Filters



When the frequency is high, $R^2 \gg X_C^2$, X_C^2 will be so small it can be ignored compared to the size of R^2 and the gain will be given by :-

$$\text{Gain}(G) = \frac{V_{OUT}}{V_{IN}} = \frac{X_C}{R} = \frac{1}{2\pi fCR}$$

which will be <1 therefore high frequency signals will be suppressed.

But what happens to mid range frequencies?

At the mid range of frequencies the value of X_C will eventually become equal to R at this point we have reached the transition between the two extremes, and it is given a special name called the **break frequency**. The break frequency has a special symbol f_b . The formula to determine f_b is obtained from equating the resistance with the reactance of the capacitor.

At the break frequency, f_b

$$\begin{aligned} X_C &= R \\ \frac{1}{2\pi f_b C} &= R \\ f_b &= \frac{1}{2\pi RC} \end{aligned}$$

Substituting $X_C=R$ in the formula for Gain we get:

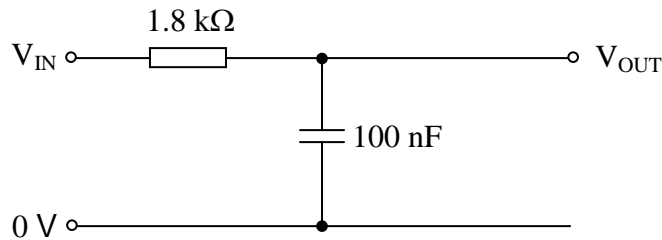
$$\text{Gain} = \frac{V_{OUT}}{V_{IN}} = \frac{X_C}{\sqrt{X_C^2 + X_C^2}} = \frac{X_C}{\sqrt{2X_C^2}} = \frac{X_C}{\sqrt{2} \times X_C} = \frac{1}{\sqrt{2}}$$

therefore

$$V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN} = 0.707 \times V_{IN}$$

At the break frequency then V_{OUT} will be 0.7 of V_{IN} . Perhaps you can remember dealing with this when you looked at the bandwidth of amplifiers in ET1.

Worked Example: Consider the following circuit:



- i) calculate the reactance of the capacitor at 10Hz, 100Hz, 1kHz, 10kHz and 100kHz.
- ii) calculate the output voltage at each of these frequencies.
- iii) calculate the break frequency of this circuit.
- iv) calculate V_{OUT} at the break frequency.
- v) plot a graph of output voltage against frequency on log graph paper.

Solution:

- i) calculate the reactance of the capacitor at 10Hz, 100Hz, 1kHz, 10kHz and 100kHz.

$$\text{At 10 Hz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 10 \times 100 \times 10^{-9}} = 159,154\Omega$$

$$\text{At 100 Hz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 100 \times 100 \times 10^{-9}} = 15,915\Omega$$

$$\text{At 1 kHz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 1000 \times 100 \times 10^{-9}} = 1,591\Omega$$

$$\text{At 10 kHz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 10 \times 10^3 \times 100 \times 10^{-9}} = 159\Omega$$

$$\text{At 100 kHz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 100 \times 10^3 \times 100 \times 10^{-9}} = 15.9\Omega$$

Topic 4.2.2 - Passive R-C Filters



ii) calculate the output voltage at each of these frequencies.

$$\text{At 10 Hz: } V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{159154}{\sqrt{1800^2 + 159154^2}} \times 10 = 9.999V$$

$$\text{At 100 Hz: } V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{15915}{\sqrt{1800^2 + 15915^2}} \times 10 = 9.936V$$

$$\text{At 1 kHz: } V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{1591}{\sqrt{1800^2 + 1591^2}} \times 10 = 6.622V$$

$$\text{At 10 kHz: } V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{159}{\sqrt{1800^2 + 159^2}} \times 10 = 0.879V$$

$$\text{At 100 kHz: } V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{15.9}{\sqrt{1800^2 + 15.9^2}} \times 10 = 0.088V$$

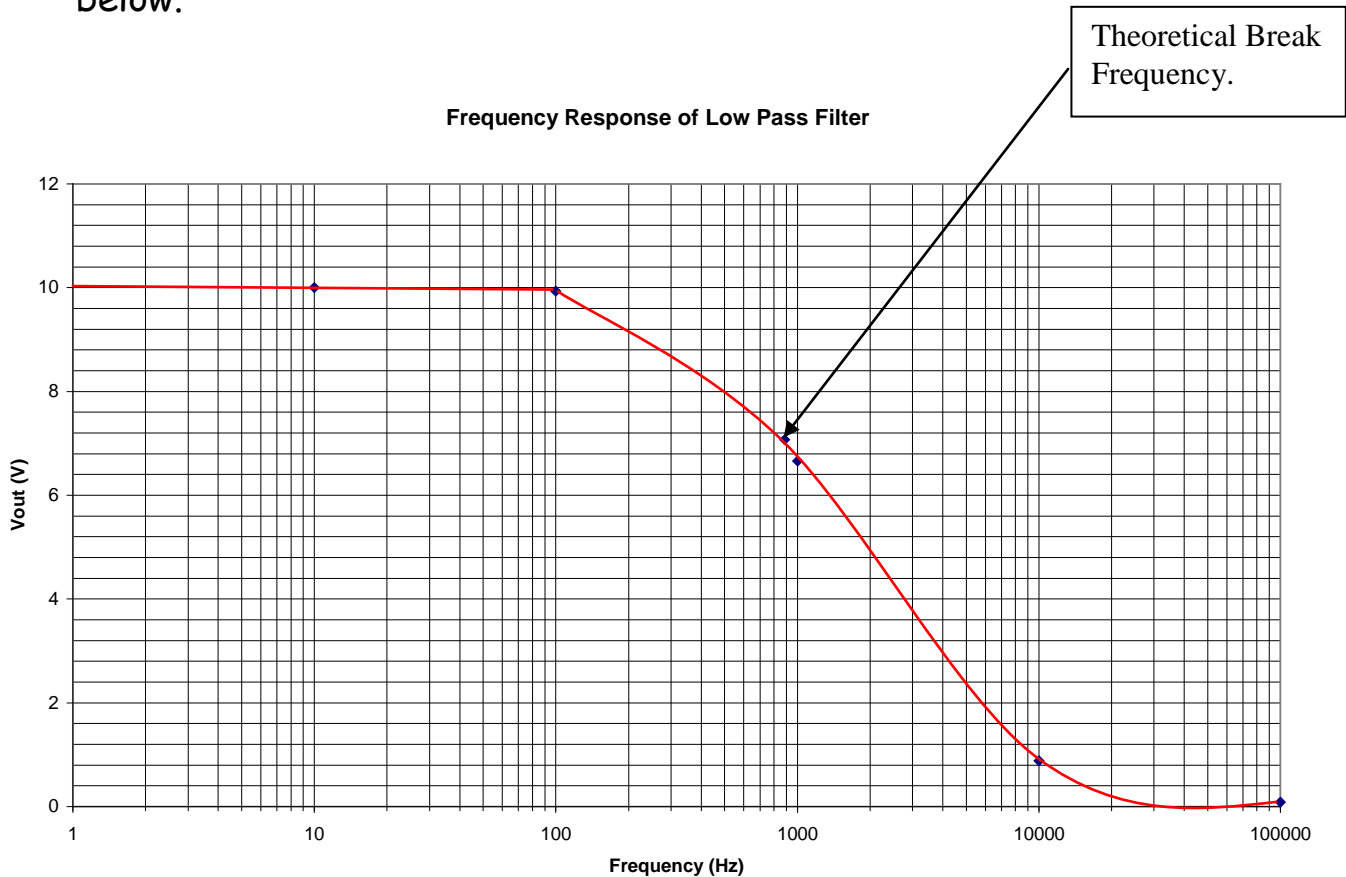
iii) calculate the break frequency of this circuit.

$$f_b = \frac{1}{2\pi RC}$$
$$= \frac{1}{2 \times \pi \times 1800 \times 100 \times 10^{-9}} = 884.19Hz$$

iv) calculate V_{OUT} at the break frequency.

$$V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN}$$
$$= 0.707 \times 10$$
$$= 7.07V$$

- v) plot a graph of output voltage against frequency on log graph paper below.

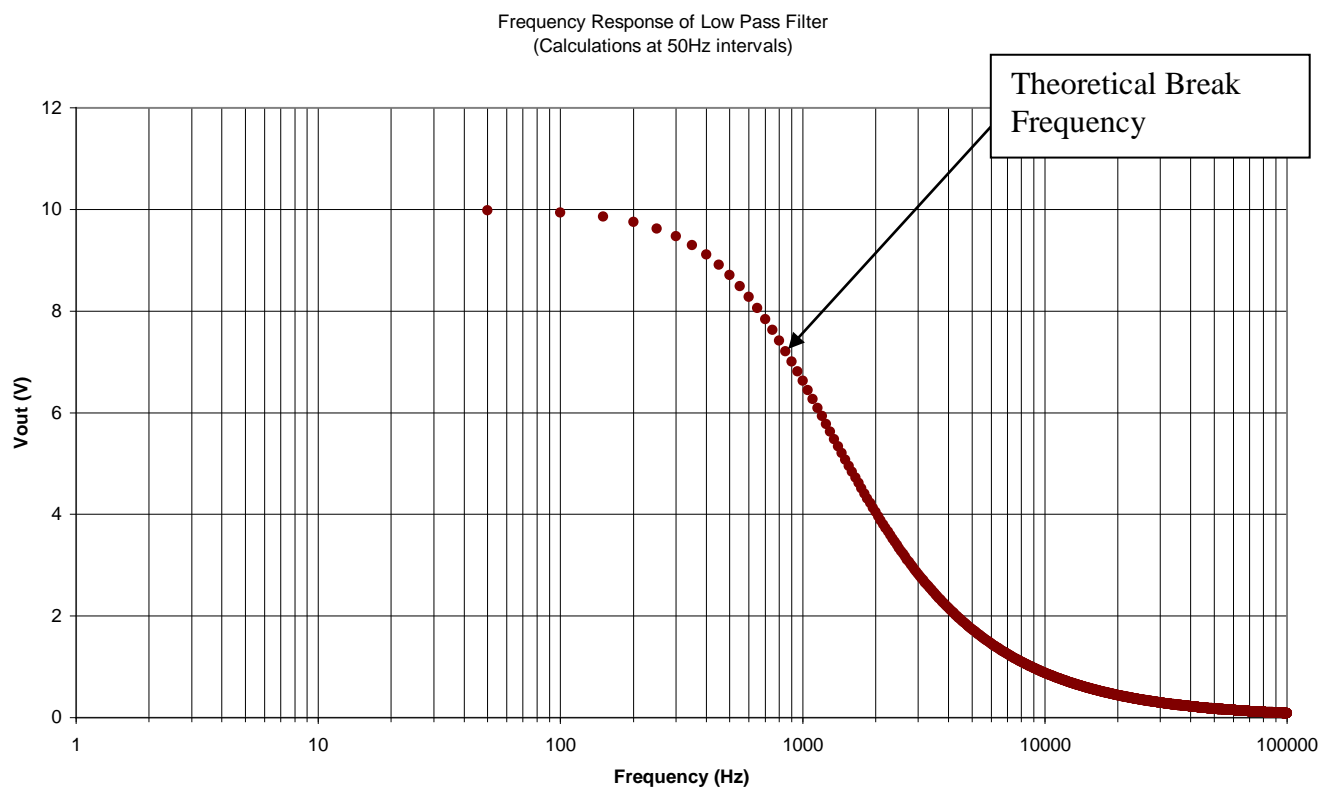


Note that the shape of the graph is generally curved, but it is difficult to see sometimes exactly the path of the curve through the data points because there are so few.

The way to improve on this is to use a computer running a program like excel to perform many calculations for us at smaller increases in frequency, and then plot the graph.

If the data for the same circuit is calculated at intervals of 50 Hz all the way up to 100,000 Hz we get the following graph.

Topic 4.2.2 - Passive R-C Filters

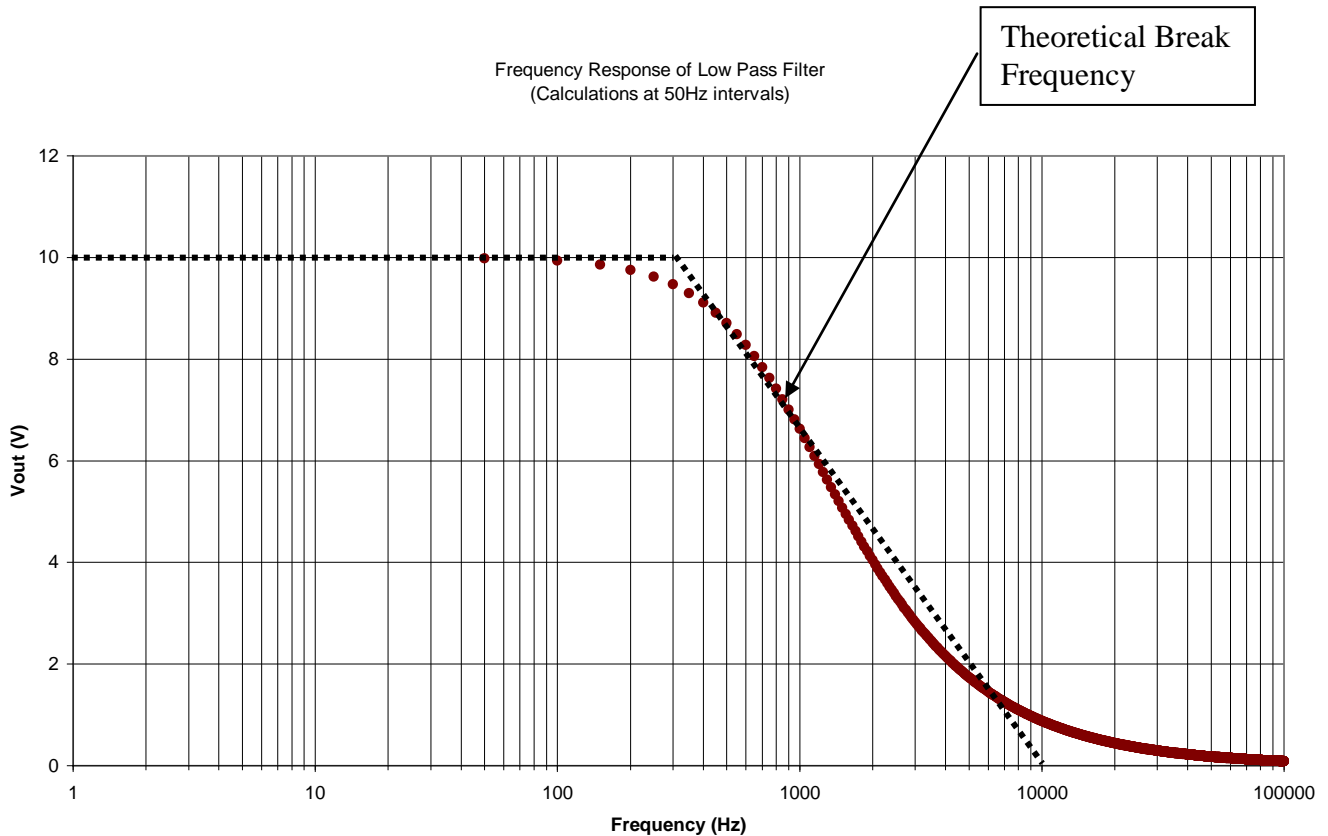


We can now see the shape of the graph much more clearly, but the response is not exactly as we discussed in the 'ideal' case. This is because we are dealing with real component values and a real circuit. The reactance of the capacitor is changing all the time as frequency changes, it doesn't suddenly switch from being high to low, and this causes the rolling over of the graph as it approaches the break frequency point.

If we compare the graph to the ideal low pass filter characteristic covered in our previous section we can see that there are several key differences in reality. (i) there is a roll off in gain as the break frequency is approached, (ii) the gain decreases slowly over a range of frequencies i.e. there is not a vertical drop at the break frequency. (iii) there is a small output voltage even at high frequencies.

You might think that it is not very effective given all these deviations from what is expected, however, in practice it works quite well for such a simple circuit.

We can simplify the graph if we only want to represent the approximate characteristic of the filter by adding straight lines to the graph as shown by the dashed lines on the graph below:



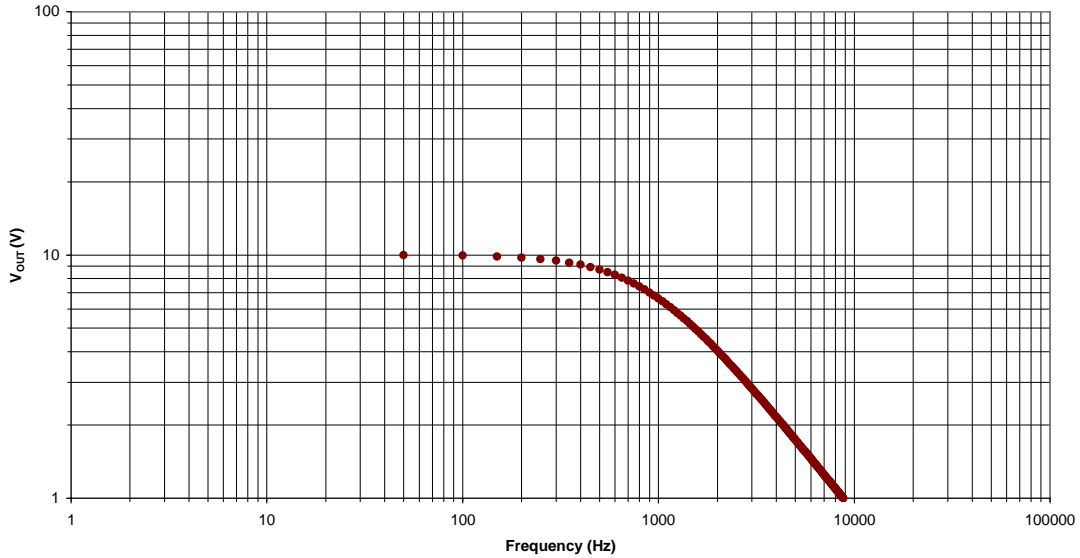
It would be acceptable in the examination to use this straight line approximation, as it would be impossible to calculate all of the points needed to produce a really smooth curve to show the actual response.

This does not mean that you don't have to be able to calculate the exact output at a specific frequency, this is a perfectly valid examination question, as is calculating the break frequency, it is just that you will not be expected to calculate hundreds of points to draw the graph accurately.

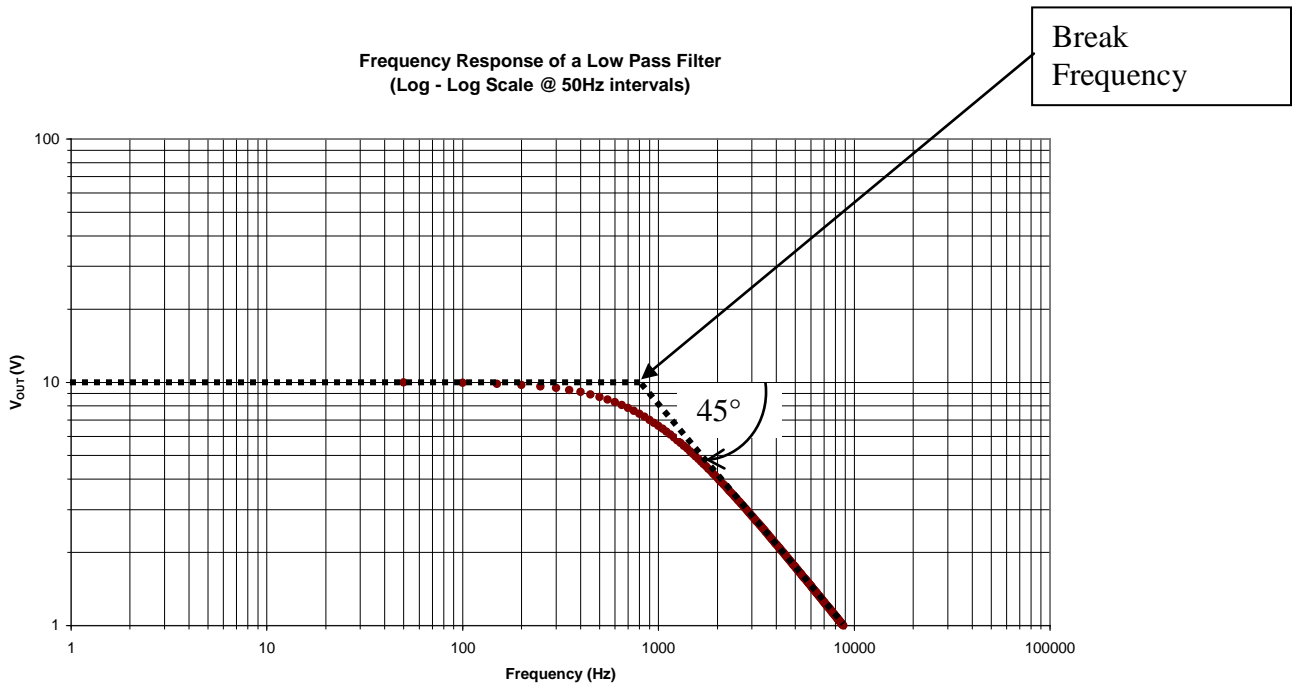
Topic 4.2.2 - Passive R-C Filters

A more accurate plot can be obtained by using log-log paper, as shown below:

Frequency Response of a Low Pass Filter
(Log - Log Scale @ 50Hz intervals)



Again it is possible to make a straight line approximation for this graph, but it is easier to draw the lines as the roll off falls at 45° as shown below.

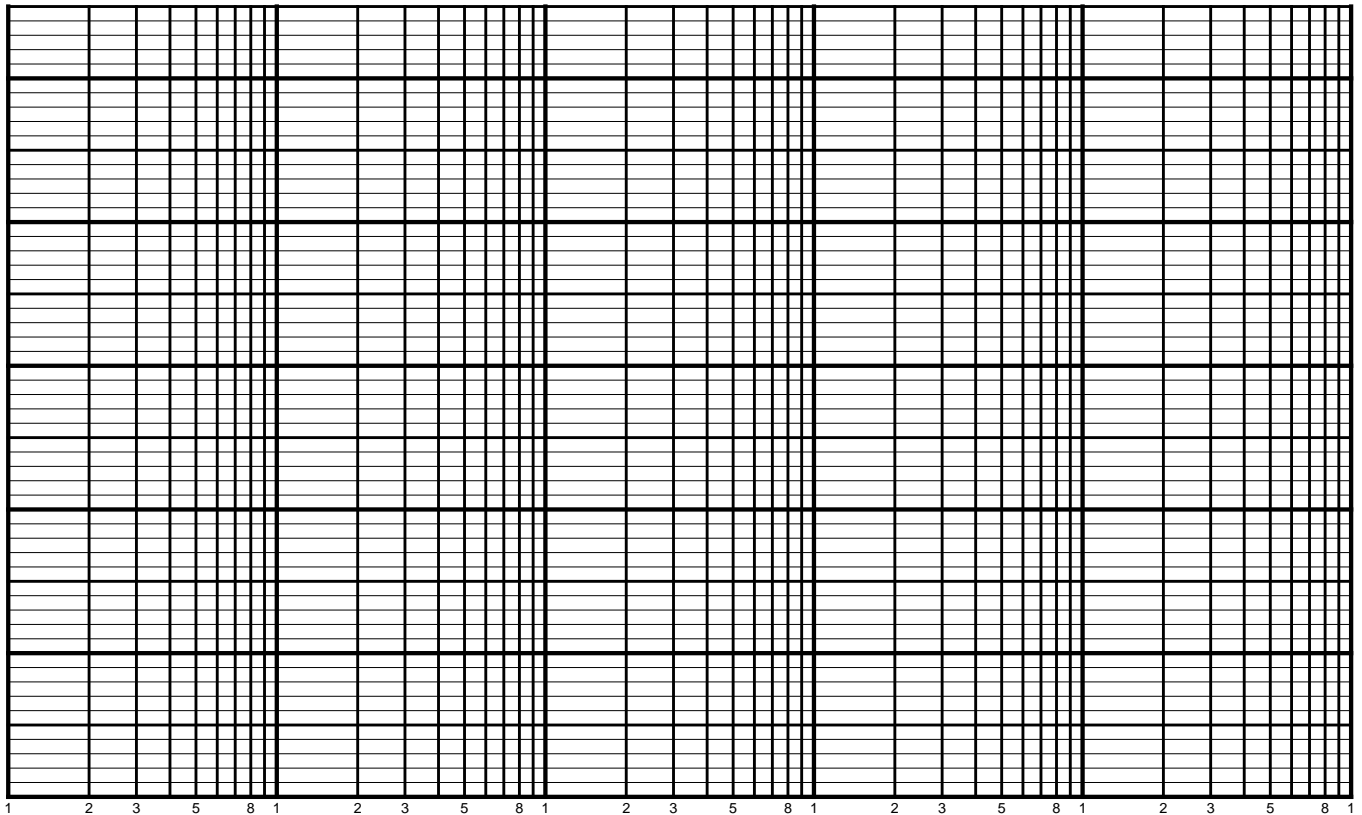


Topic 4.2.2 - Passive R-C Filters



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Graph paper to plot frequency response.

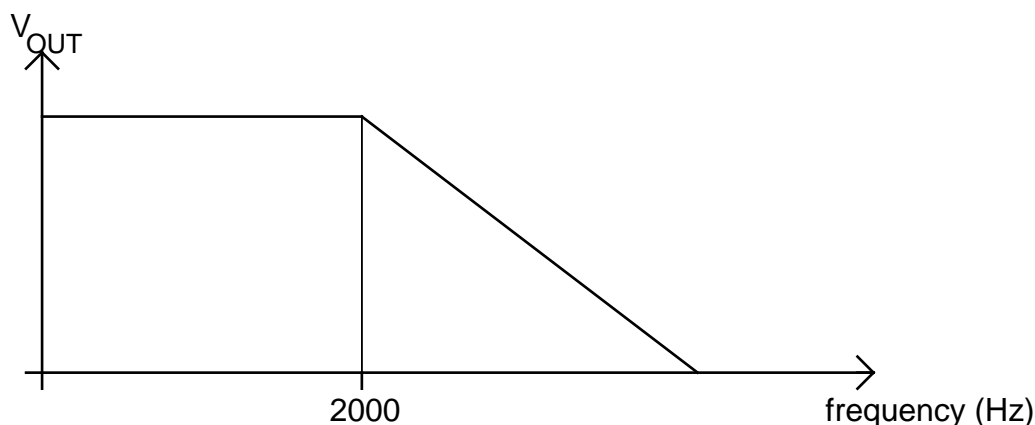


We have seen how to draw and sketch the characteristic when we are given a circuit diagram, but what if we are given the characteristic, and asked to design the circuit?

In many respects this is a much easier problem to solve. Lets look at an example to see why.

Worked Example:

Determine the value of resistor and capacitor required for a low pass filter to produce the following characteristic.



In this case we are given the break frequency, at 2 kHz from the graph. Using the formula for break frequency we get:

$$f_b = \frac{1}{2\pi RC}$$

$$2000 = \frac{1}{2\pi RC}$$

We need to know either the value of R , or C in order to solve this equation. Sometimes in an examination you are given a few to choose from, but if you are not given any you just have to choose your own value for one and then work out the corresponding value. In this case as we have not been given any, we will try using a 22nF capacitor. The formula can now be rearranged to find R as follows:

$$f_b = \frac{1}{2\pi RC}$$

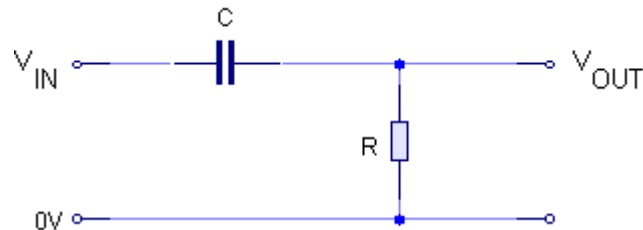
$$2000 = \frac{1}{2\pi RC}$$

$$R = \frac{1}{2 \times \pi \times 2000 \times 22 \times 10^{-9}} = 3617\Omega$$

A resistor of 3617Ω , would give the exact breakpoint required, but is not available in the range of preferred values, a $3.3k\Omega$ or $3.9k\Omega$ would be suitable, especially given the fact that the characteristic produced will be a curve anyway.

ii) A Simple High Pass Filter.

A high pass filter (HPF) is used to remove low frequency signals from a signal spectrum. The circuit is again very straightforward, but should not be confused with the Low Pass Filter.



The circuit now consists of a capacitor in series with a resistor. The output voltage is taken across the resistor as shown.

Again we consider the circuit to be a potential divider, and write down the formula for output voltage as we did before; the only difference is that the output is taken across the resistor instead of the capacitor. i.e.

$$V_{OUT} = \frac{V_{IN}}{Z} \times R$$

$$= \frac{V_{IN}}{\sqrt{R^2 + X_C^2}} \times R$$

This can be re-arranged to look at the 'gain' of the circuit. From the previous work you will remember that gain is defined as $\frac{V_{OUT}}{V_{IN}}$. i.e.

$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{\sqrt{R^2 + X_C^2}}$$

Taking a look at the two extremes, i.e. very low and very high frequencies.

When the frequency is high, $R^2 \gg X_C^2$, X_C^2 will be so small it can be ignored compared to the size of R^2 and the gain will be near 1. i.e. no change.

Topic 4.2.2 - Passive R-C Filters



When the frequency is low, $X_C^2 \gg R^2$, R^2 will be so small it can be ignored compared to the size of X_C^2 and the gain will be given by :-

$$\text{Gain}(G) = \frac{V_{OUT}}{V_{IN}} = \frac{R}{X_C} = \frac{R}{\frac{1}{2\pi fC}} = 2\pi fCR$$

which will be <1 therefore low frequency signals will be suppressed (given that the size of capacitors used in these filters are in the nF range).

But what happens to mid range frequencies?

As before, at the mid range of frequencies the value of R and X_C will become equal at the **break frequency**, f_b . The formula to determine f_b is obtained from equating the resistance with the reactance of the capacitor.

At the break frequency, f_b

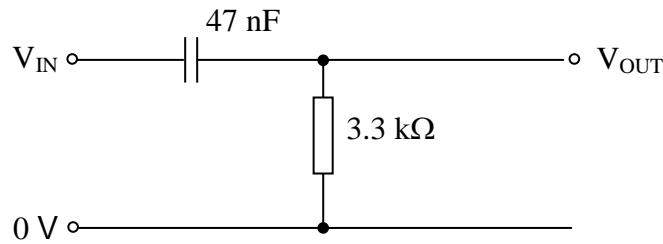
$$\begin{aligned}X_C &= R \\ \frac{1}{2\pi f_b C} &= R \\ f_b &= \frac{1}{2\pi RC}\end{aligned}$$

Note : The formula is identical to that for the low pass filter.

In a similar way the value of V_{OUT} at the break frequency will be given by :

$$V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN}$$

Worked Example: Consider the following circuit:



- i) calculate the reactance of the capacitor at 10Hz, 100Hz, 1kHz, 10kHz and 100kHz.
- ii) calculate the output voltage at each of these frequencies.
- iii) calculate the break frequency of this circuit.
- iv) calculate V_{OUT} at the break frequency.
- v) plot a graph of output voltage against frequency on log graph paper.

Solution :

- i) calculate the reactance of the capacitor at 10Hz, 100Hz, 1kHz, 10kHz and 100kHz.

$$\text{At 10 Hz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 10 \times 47 \times 10^{-9}} = 338,627\Omega$$

$$\text{At 100 Hz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 100 \times 47 \times 10^{-9}} = 33,863\Omega$$

$$\text{At 1 kHz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 1000 \times 47 \times 10^{-9}} = 3,386\Omega$$

$$\text{At 10 kHz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 10 \times 10^3 \times 47 \times 10^{-9}} = 339\Omega$$

$$\text{At 100 kHz: } X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 100 \times 10^3 \times 47 \times 10^{-9}} = 33.9\Omega$$

Topic 4.2.2 - Passive R-C Filters



ii) calculate the output voltage at each of these frequencies.

$$\text{At 10 Hz: } V_{OUT} = \frac{R}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 338627^2}} \times 10 = 0.097V$$

$$\text{At 100 Hz: } V_{OUT} = \frac{R}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 33863^2}} \times 10 = 0.969V$$

$$\text{At 1 kHz: } V_{OUT} = \frac{R}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 3386^2}} \times 10 = 6.979V$$

$$\text{At 10 kHz: } V_{OUT} = \frac{R}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 339^2}} \times 10 = 9.947V$$

$$\text{At 100 kHz: } V_{OUT} = \frac{R}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 33.9^2}} \times 10 = 9.999V$$

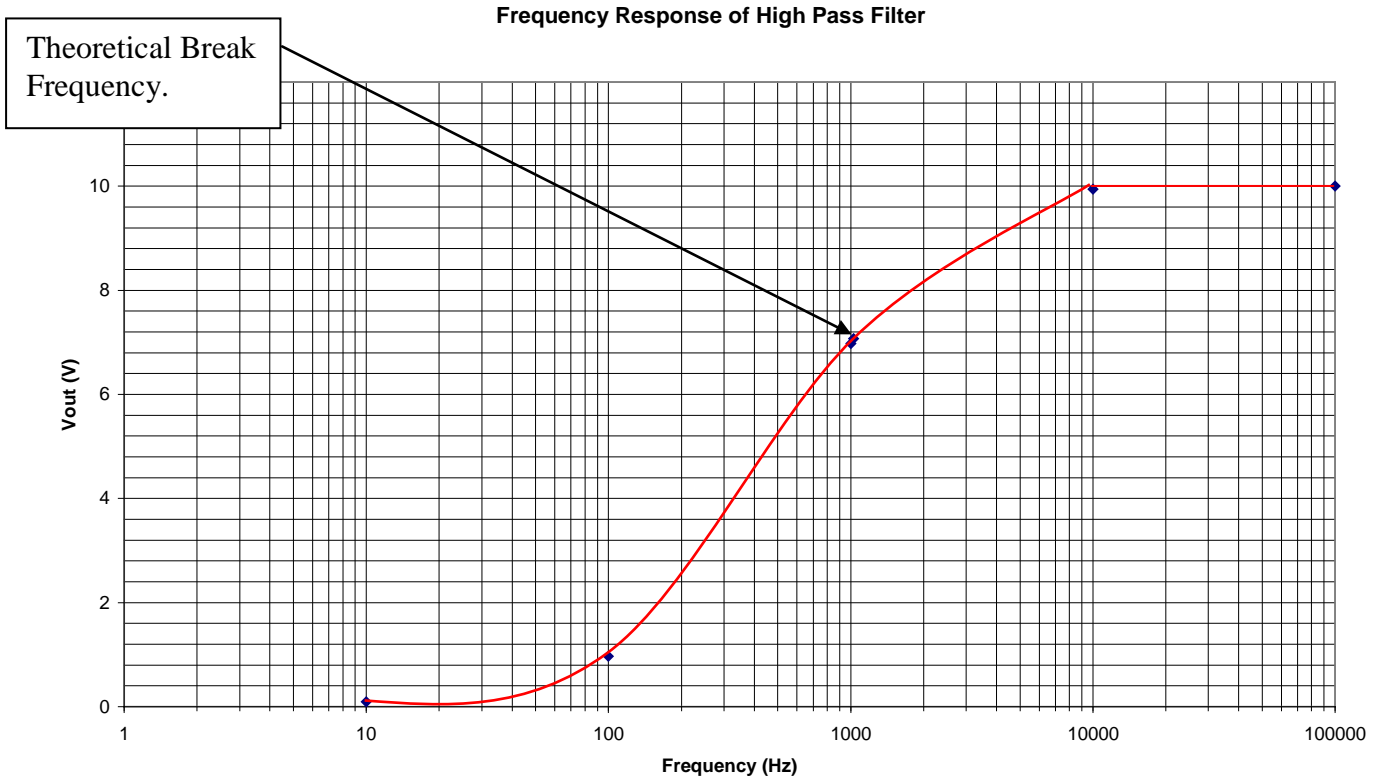
iii) calculate the break frequency of this circuit.

$$f_b = \frac{1}{2\pi RC}$$
$$= \frac{1}{2 \times \pi \times 3300 \times 47 \times 10^{-9}} = 1026.14Hz$$

iv) calculate V_{OUT} at the break frequency.

$$V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN}$$
$$= 0.707 \times 10$$
$$= 7.07V$$

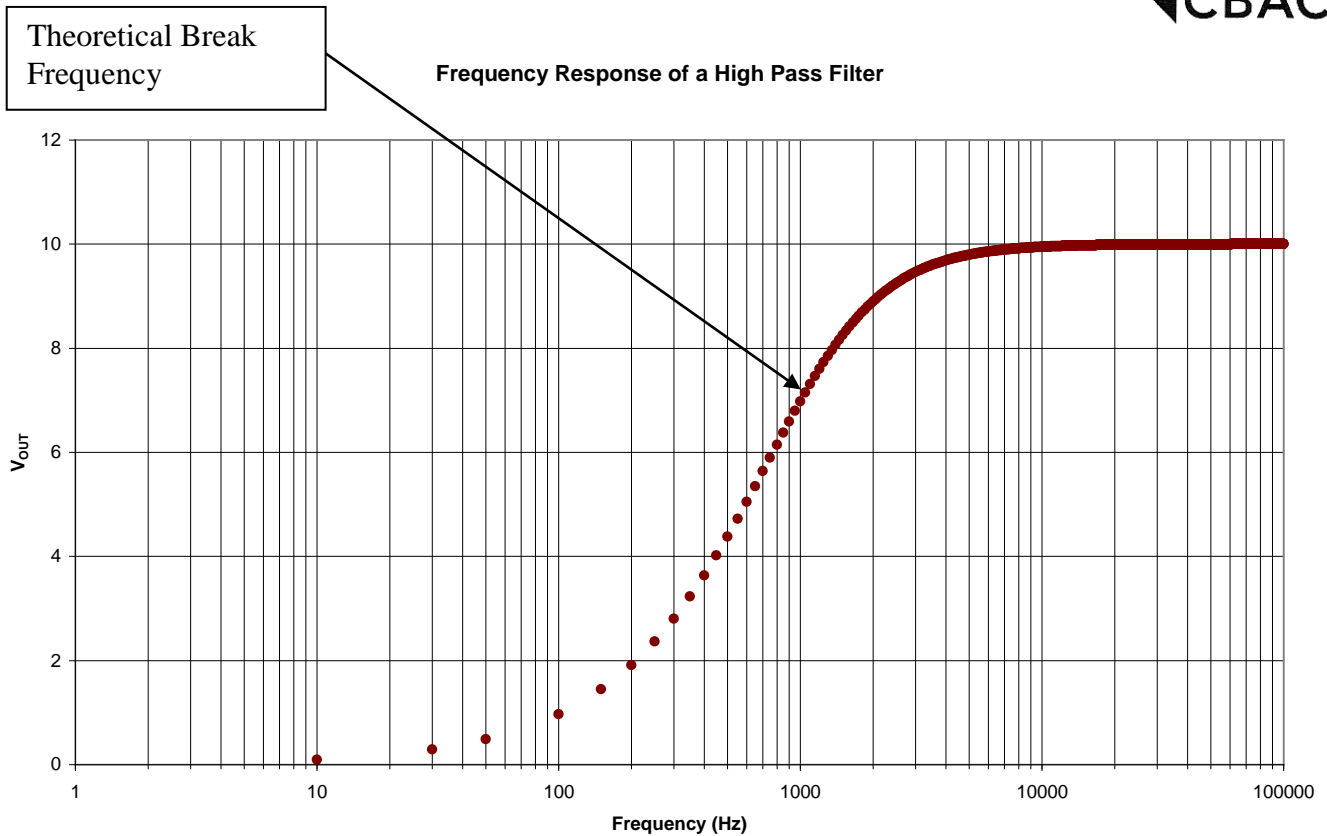
- iv) plot a graph of output voltage against frequency on log graph paper below.



Note that once again the shape of the graph is generally curved, but it is difficult to see sometimes exactly the path of the curve through the data points because there are so few.

Again we can use Excel to make a better attempt at this by calculating all of the data for the same circuit in intervals of 50 Hz all the way up to 100,000 Hz we get the following graph.

Topic 4.2.2 - Passive R-C Filters

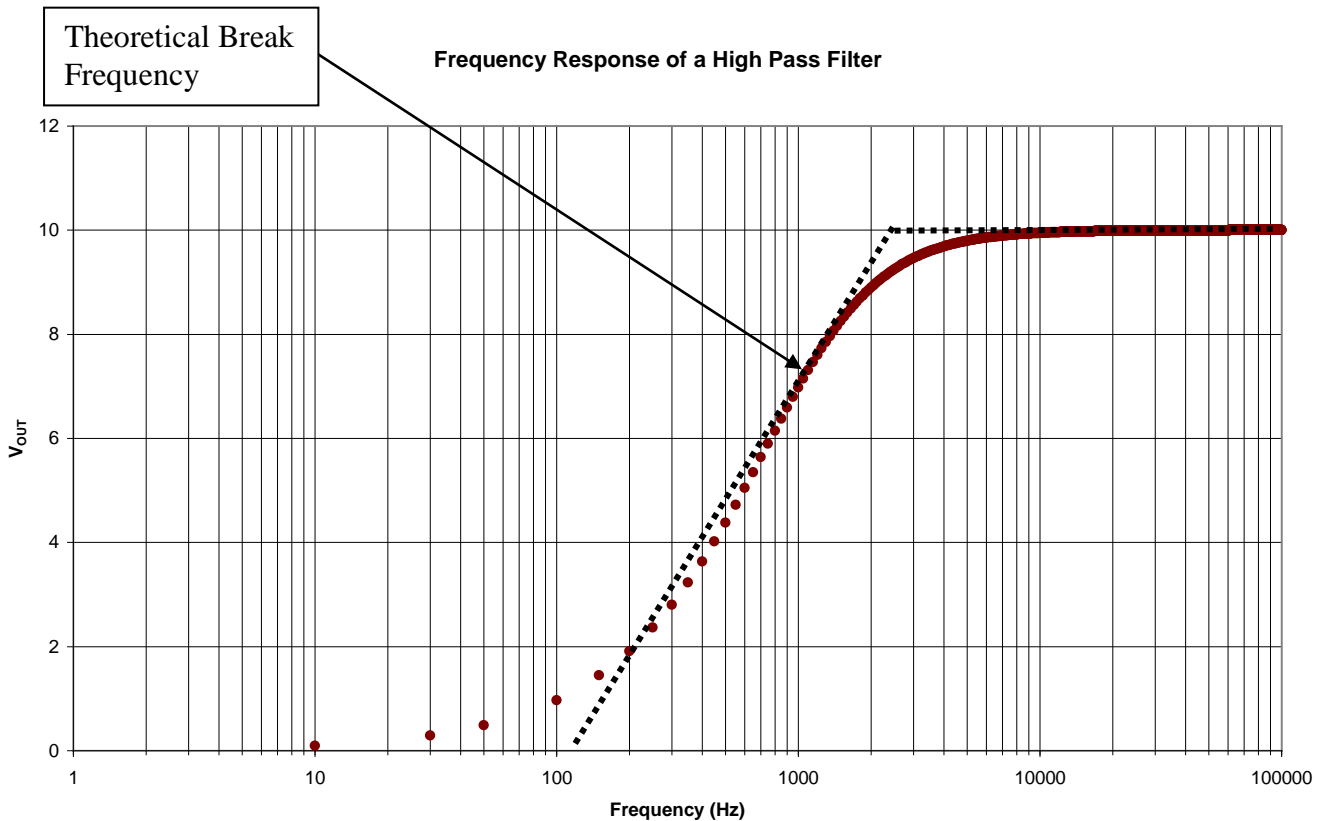


We can now see the shape of the graph much more clearly.

If we compare the graph to the ideal high pass filter characteristic covered in our previous section we can see that there are several key differences in reality. (i) there is a roll off in gain as the break frequency is approached, (ii) the gain decreases slowly over a range of frequencies i.e. there is not a vertical drop at the break frequency. (iii) there is a small output voltage even at low frequencies.

You might think that it is not very effective given all these deviations from what is expected, however, in practice it works quite well for such a simple circuit.

We can simplify the graph if we only want to represent the approximate characteristic of the filter by adding straight lines to the graph as shown by the dashed lines on the graph below:



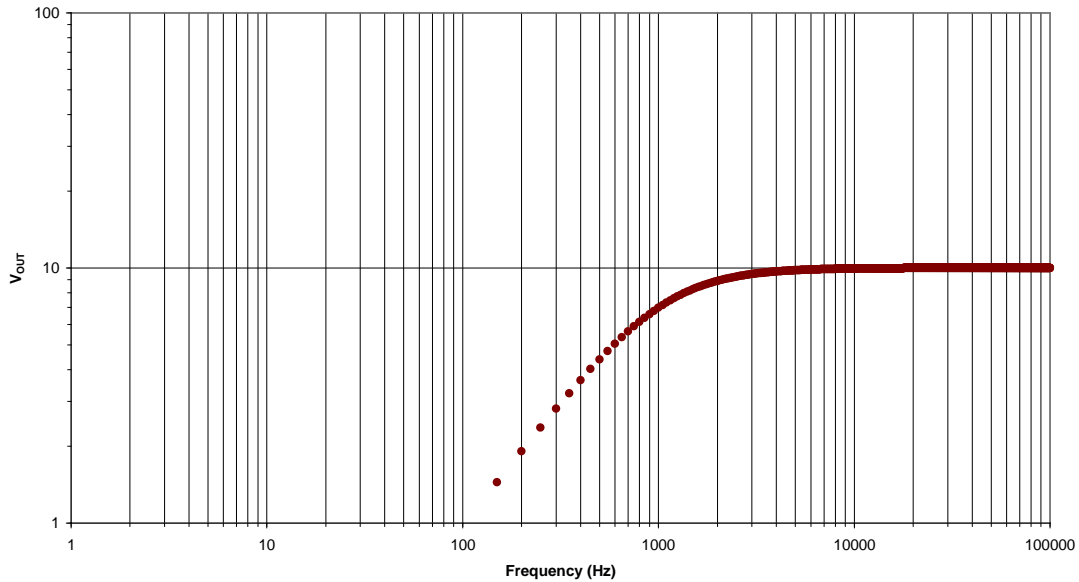
It would be acceptable in the examination to use this straight line approximation, as it would be impossible to calculate all of the points needed to produce a really smooth curve to show the actual response.

This does not mean that you don't have to be able to calculate the exact output at a specific frequency, this is a perfectly valid examination question, as calculating the break frequency, it is just that you will not be expected to calculate hundreds of points to draw the graph accurately.

Topic 4.2.2 - Passive R-C Filters

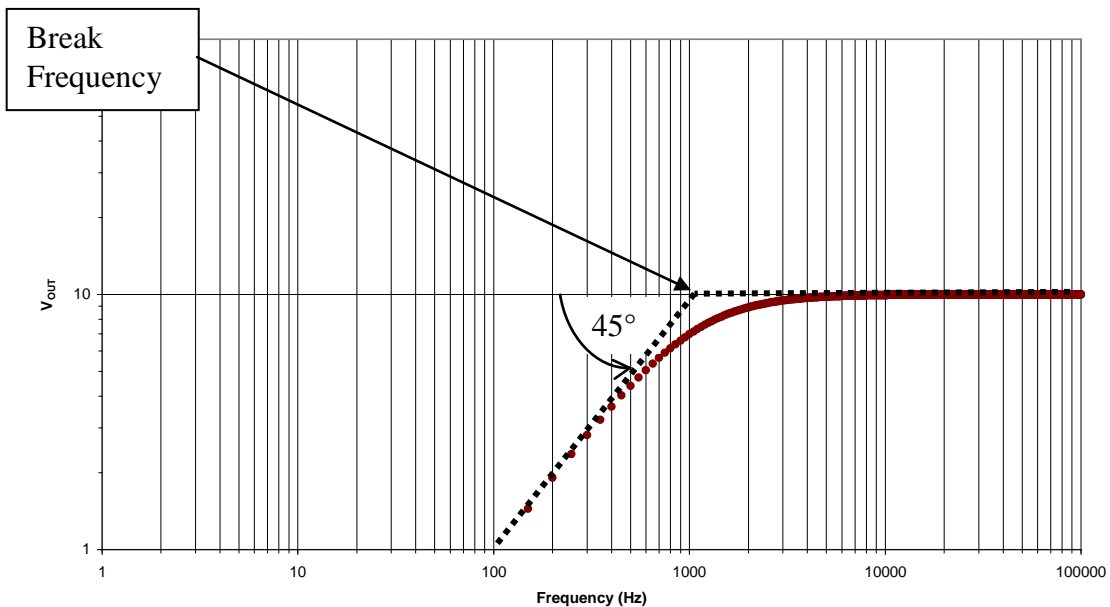
Once again a more accurate and appropriate way to draw this graph would be on log - log graph paper as shown below.

Frequency Response of a High Pass Filter



Again it is possible to make a straight line approximation for this graph, but it is easier to draw the lines as the roll off falls at 45° as shown below.

Frequency Response of a High Pass Filter

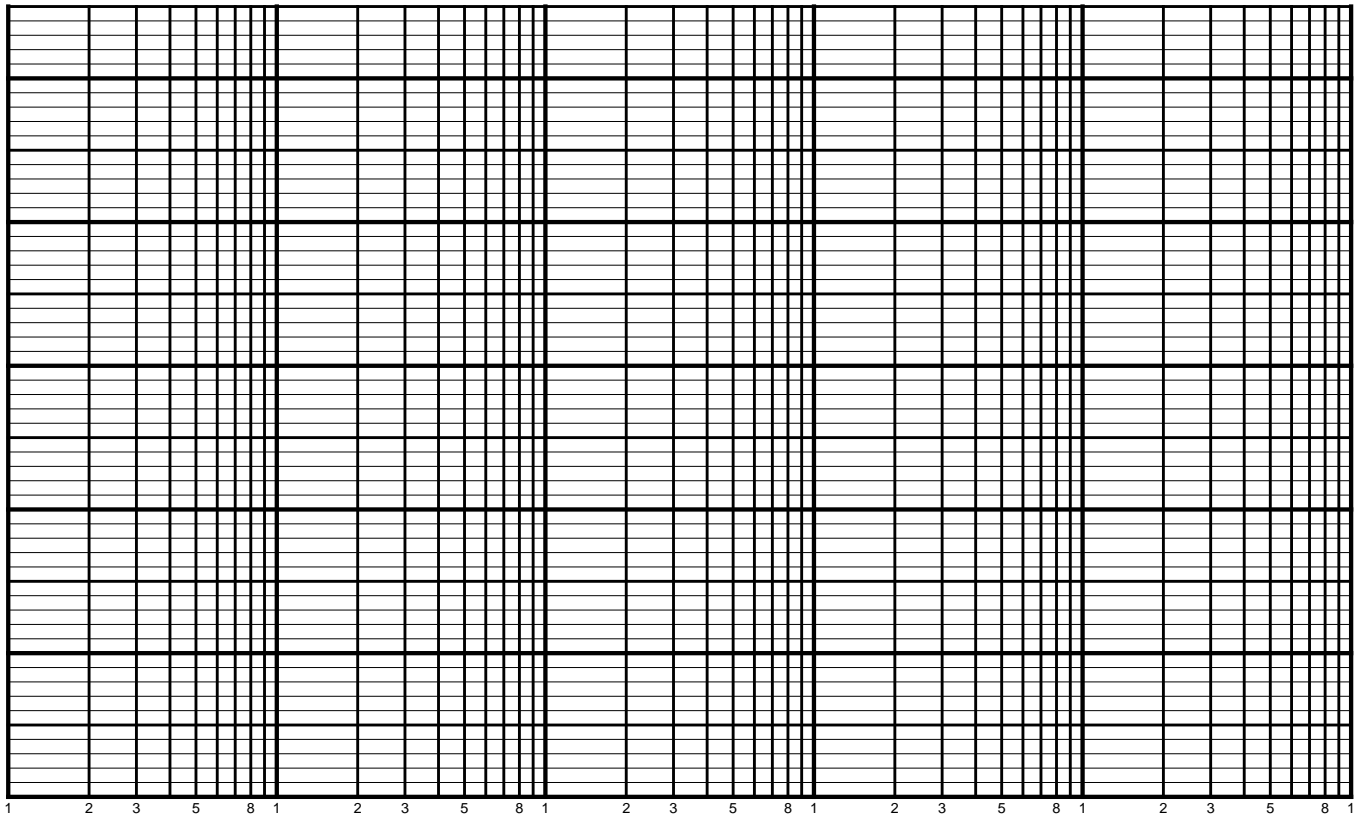


Topic 4.2.2 - Passive R-C Filters



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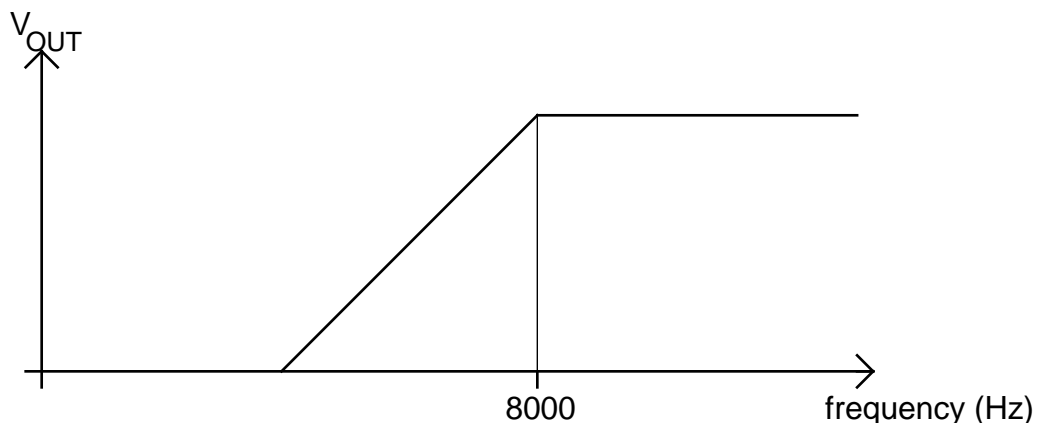
Graph paper to plot frequency response.



Time now to look at a design question where the characteristic is given and component values have to be calculated.

Worked Example:

Design a high pass filter to produce the following characteristic, you have available the following capacitors, $10\mu\text{F}$, 22nF , and 0.47pF to choose from. Draw the circuit diagram of the completed filter. How would you expect the actual performance of your filter to compare to the profile shown ?



Topic 4.2.2 - Passive R-C Filters



In this case we are given the break frequency, at 8 kHz from the graph. Using the formula for break frequency we get:

$$f_b = \frac{1}{2\pi RC}$$
$$8000 = \frac{1}{2\pi RC}$$

We are also told that we have a choice of three capacitors available $10\mu\text{F}$, 22nF and 0.47pF . How do you decide which to use? Well this comes with experience, but if you really have no idea, try calculating the value of R with each one. Assume the E24 series of resistors is available.

With $C=10\mu\text{F}$

$$f_b = \frac{1}{2\pi RC}$$
$$8000 = \frac{1}{2\pi RC}$$
$$R = \frac{1}{2 \times \pi \times 8000 \times 10 \times 10^{-6}} = 1.98\Omega \approx 2\Omega$$

With $C=22\text{nF}$

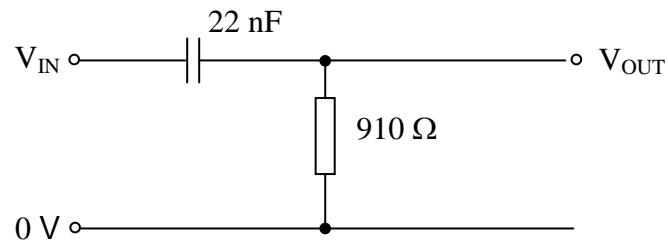
$$f_b = \frac{1}{2\pi RC}$$
$$8000 = \frac{1}{2\pi RC}$$
$$R = \frac{1}{2 \times \pi \times 8000 \times 22 \times 10^{-9}} = 904.28\Omega \approx 910\Omega$$

With $C=0.47\text{pF}$

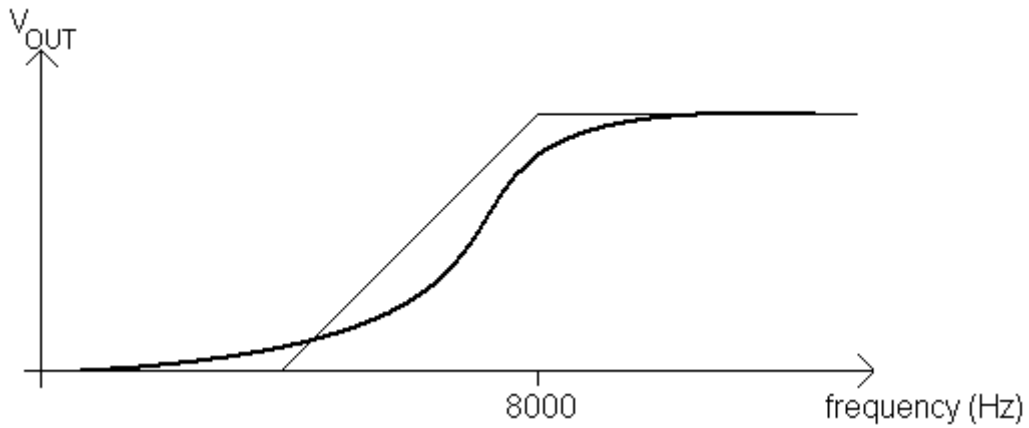
$$f_b = \frac{1}{2\pi RC}$$
$$8000 = \frac{1}{2\pi RC}$$
$$R = \frac{1}{2 \times \pi \times 8000 \times 0.47 \times 10^{-12}} = 42328442\Omega \approx 43\text{M}\Omega$$

Having calculated the three resistors, the 2Ω resistor is unsuitable as it has too low a resistance drawing an excess current from the source and $43\text{M}\Omega$ is beyond the E24 series which stops at $10\text{M}\Omega$. Therefore 910Ω would be the most acceptable resistor to use here from a practical point of view.

The completed circuit will therefore look like this:



In practice the response of the circuit above will be more curved and will be more like the following:



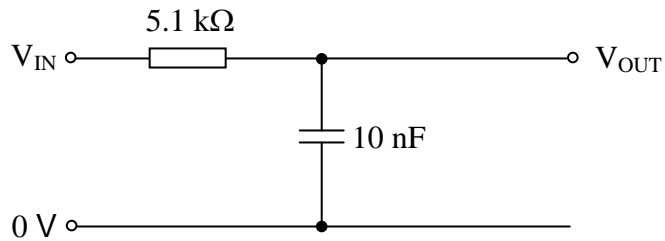
Now for some practice examination questions.

Topic 4.2.2 - Passive R-C Filters



Examination Style Questions.

1. The following circuit is to be used as a filter.



a. What is the name of this type of filter ? [1]

b. Calculate the reactance of the capacitor at 100 Hz.

 [2]

c. Estimate the reactance of the capacitor at 10 kHz.
 [1]

d. Calculate the break frequency for this filter.

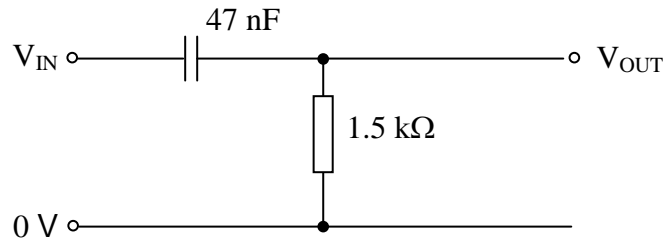
 [2]

e. Sketch the characteristic of this filter labelling all critical values.



[2]

2. The following circuit is to be used as a filter.



a. What is the name of this type of filter ? [1]

b. Calculate the reactance of the capacitor at 1000 Hz.
.....
.....
..... [2]

c. What is the impedance of the circuit at 1000 Hz.
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..... [2]

d. Calculate the output voltage if $V_{IN} = 10V$ at 1000 Hz.
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..... [1]

e. Calculate the break frequency for this filter.
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..... [2]

Topic 4.2.2 - Passive R-C Filters

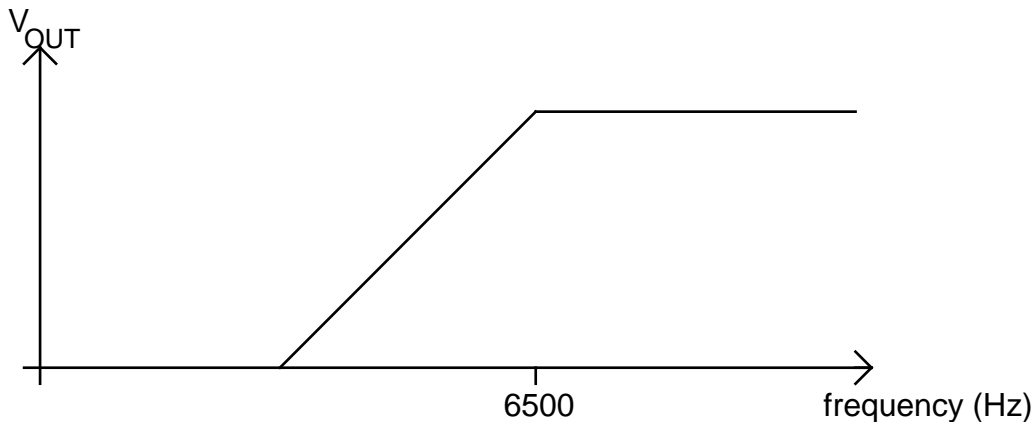


- f. Sketch the characteristic of this filter labelling all critical values.

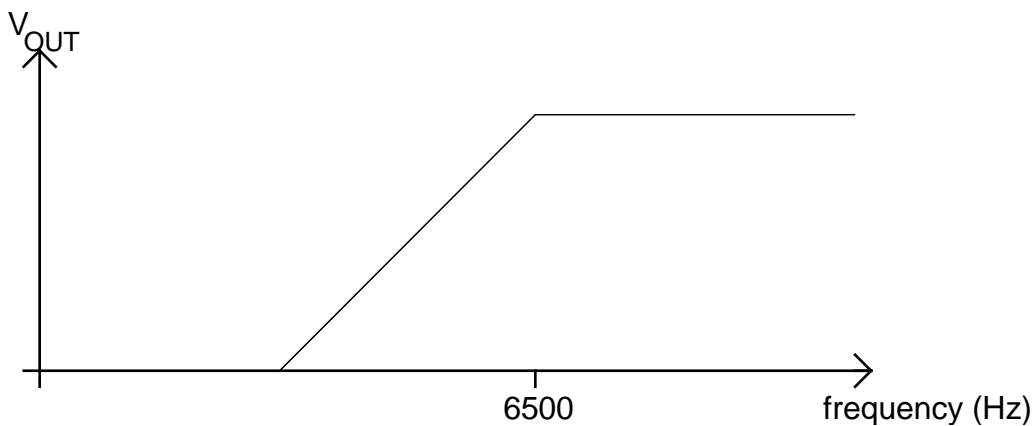


[2]

3. The ideal frequency response for a filter is shown below.



- a) This filter is built using a simple RC circuit. Sketch the response you are likely to obtain from the simple RC filter circuit clearly showing how it would **differ** from the ideal response which is shown as a dotted line.



- b. You have available the following capacitors, $1\mu\text{F}$, 47nF , and 6.8pF and the E24 series of resistors to choose from. Calculate the most suitable resistor for this filter.

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- c. Draw the circuit diagram of the completed filter, label all component values and input and output terminals. [3]

- d. Give reasons why you have chosen the components you have for your filter. [2]

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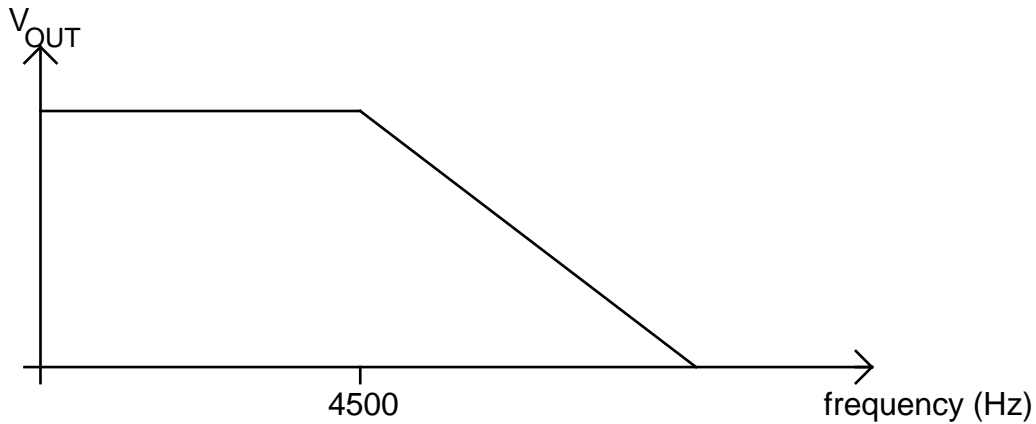
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[2]

Topic 4.2.2 - Passive R-C Filters



4. A low pass filter is required to correspond to the following characteristic. You can assume you have access to any standard value capacitors and resistors.



- a. What is the break frequency of this filter ? [1]
- b. Draw the circuit diagram of a low pass filter.

- c. Determine the value of the components required to realise the function of the low pass filter identified in the characteristic. [2]

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Self Evaluation Review

Learning Objectives	My personal review of these objectives:		
			
recognise, analyse, and sketch characteristics for a low pass and a high pass filter;			
design circuits to act as low pass or high pass filters;			
select and use the formula $X_c = \frac{1}{2\pi fC}$;			
understand the significance of the term impedance, and that it is a function of X_c , and R in an R-C circuit;			
select and use the formula $Z = \sqrt{R^2 + X_c^2}$ to calculate the impedance of a series R-C circuit.			
define and calculate the break frequency, selecting and using the formula $f_b = \frac{1}{2\pi RC}$;			
plot and interpret graphs showing the frequency response of an R-C filter.			

Targets: 1.

 2.
