

Learning Objectives:

At the end of this topic you will be able to;

- ☑ sketch and recognise the resulting waveforms for a sinusoidal carrier being amplitude modulated by a single frequency audio signal;
- ☑ draw and analyse graphs to show the resulting waveform, and frequency spectrum for a sinusoidal carrier amplitude modulated by an audio signal, to a given depth of modulation, m ;
- ☑ select and use the formula:

$$m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$$

to calculate the depth of modulation for a given amplitude modulated RF signal.

Amplitude Modulation

In the introductory section we saw that a carrier without modulation can communicate very little information. The signal can therefore be considered as having two parts;

1. the information signal - this is the message we need to send and could be speech, text, or pictures.
2. the carrier - this will be the method of passing the information signal from the transmitter to receiver, usually a radio wave, microwave, light wave or electrical current.

In **Amplitude Modulation (AM)** as its name suggests, it is the amplitude of the carrier wave which becomes altered by the instantaneous value of the information signal. The following diagram illustrates this effect.

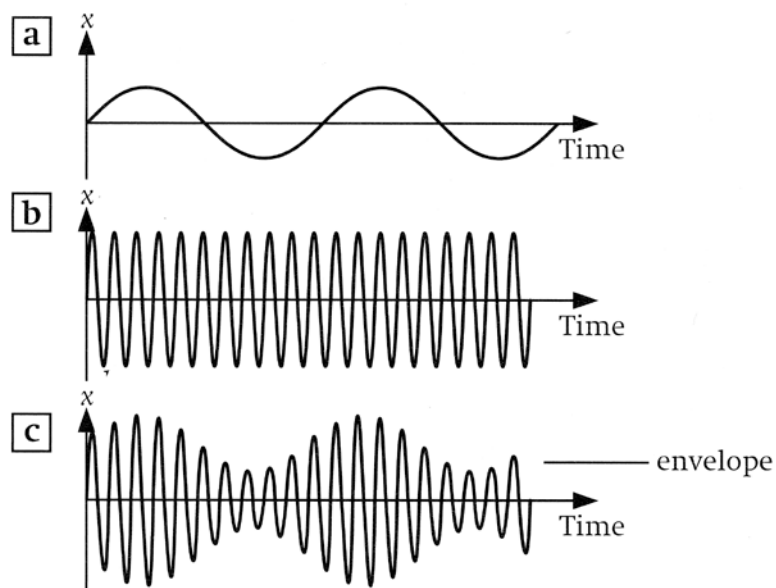


Diagram 'a' shows the information signal.

Diagram 'b' shows the unmodulated carrier signal.

Diagram 'c' shows the modulated carrier signal.

For the Enthusiast !

The exact way in which the amplitude modulated carrier is produced is quite complex and involves some advanced mathematics. The solution is provided here for those who are enthusiastic about such things.

The general mathematical formula for a sinusoidal wave is :

$$V = V_{\max} \sin 2\pi ft$$

Where V = instantaneous value of voltage, V_{\max} = maximum amplitude of the wave, f = frequency of wave and t = time.

For a carrier wave having a frequency f_c and amplitude A_c , the instantaneous value V_c can be obtained using the following equation.

$$V_c = A_c \sin 2\pi f_c t$$

The simplest information signal that can be applied will be a pure sine wave. Assume that this has a frequency f_i and amplitude A_i , the instantaneous value V_i can be obtained using the following equation.

$$V_i = A_i \sin 2\pi f_i t$$

When the carrier is amplitude modulated, the resulting wave is governed by the equation:

$$V_{AM} = (A_c + A_i \sin 2\pi f_i t) \sin 2\pi f_c t$$

This formula shows that the amplitude of the modulated carrier sine function increases above and decreases below the carrier amplitude A_c by the instantaneous value of the information signal.

Using standard mathematical rules this equation can be expanded to become:

$$V_{AM} = A_c \sin 2\pi f_c t + \frac{1}{2} A_i [\cos(2\pi f_c - 2\pi f_i)t - \cos(2\pi f_c + 2\pi f_i)t]$$

The first term represents the carrier frequency, the second is the lower side frequency and the third is the upper side frequency.

When the AM carrier wave is analysed, we find that it has three different components.

- The original carrier frequency f_c and original Amplitude A_c ;
- A wave of frequency $f_c - f_i$, called the lower side frequency;
- A wave of frequency $f_c + f_i$, called the upper side frequency;

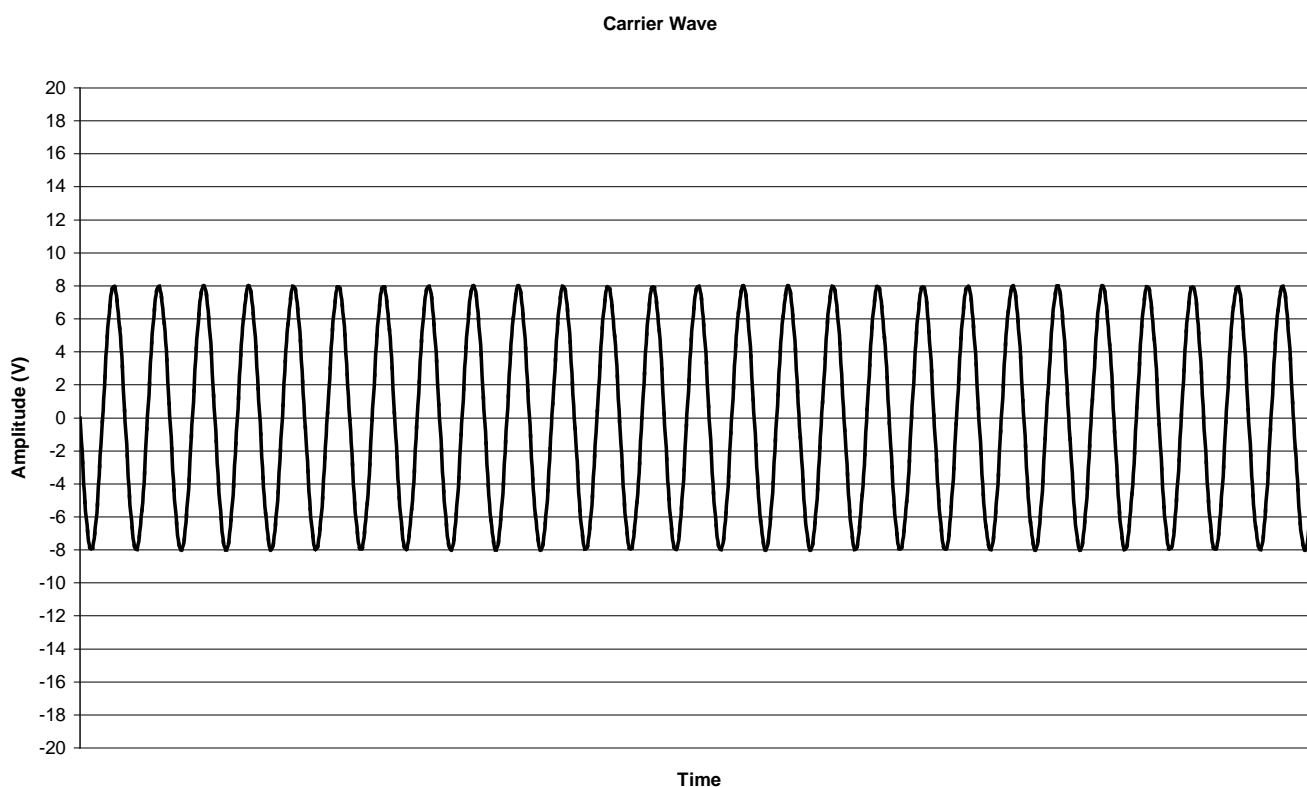
Other observations that can be made are as follows:

- The original signal frequency f_i has disappeared;
- The maximum amplitude of the AM carrier is $A_c + A_i$;
- The minimum amplitude of the AM carrier is $A_c - A_i$;
- The carrier frequency f_c must be greater than the information frequency f_i ;
- The carrier amplitude A_c must be greater than the information amplitude A_i .

Depth of Modulation.

Depth of modulation is the term given to how much the amplitude of the carrier wave is affected by the information signal. This is best illustrated by considering some examples.

To demonstrate the effect the same carrier signal will be used as shown in the following diagram. The amplitude of the carrier is 8V, and is of high frequency, no units have been added to the time axis as these are illustrative diagrams only.

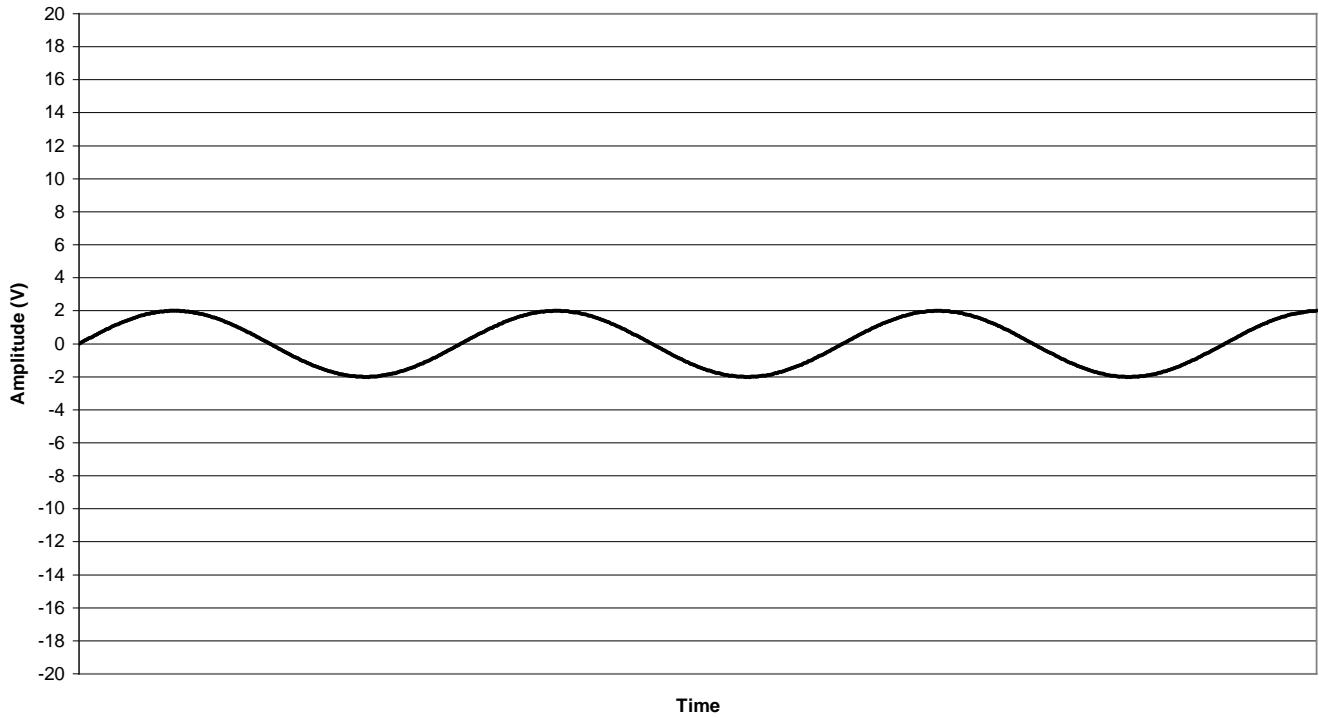


We will now add a number of different information signals, of lower frequency but with amplitudes of 2V, 4V, 6V, 8V and 10V to see the effect on the modulated signal.

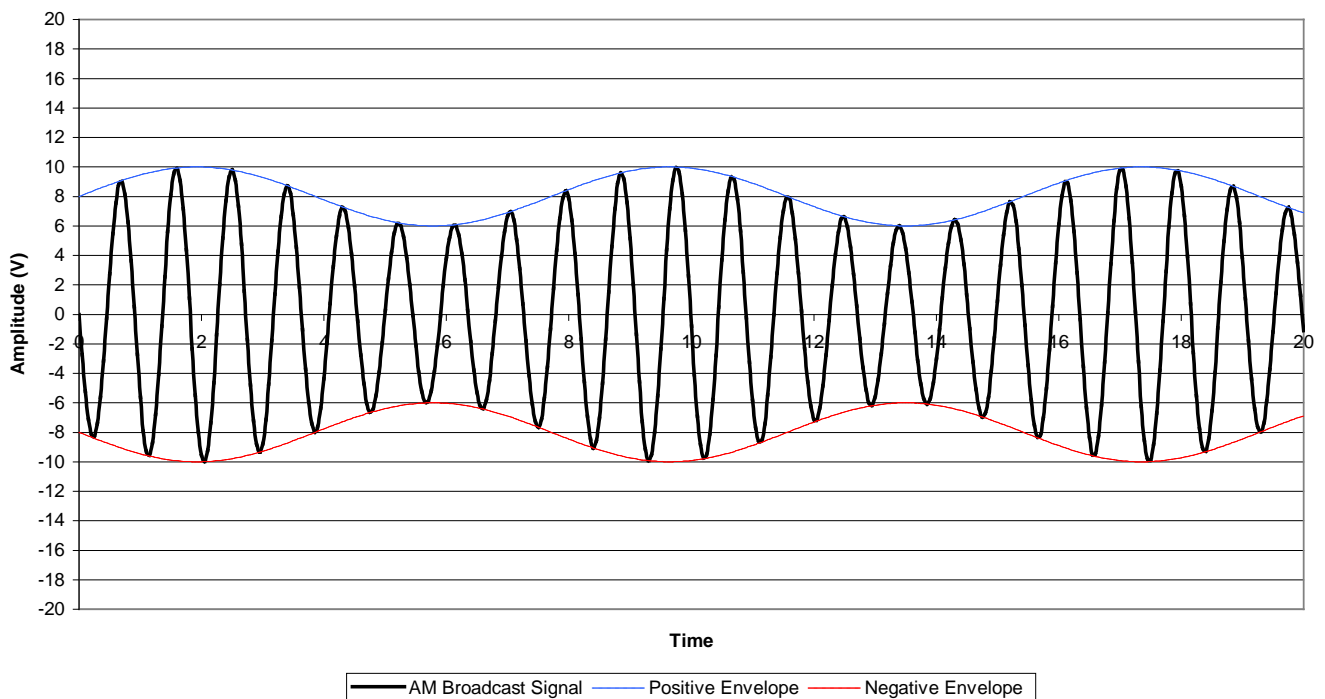
Example 1.

In this example, the information signal has an amplitude of 2V.

Information Signal Wave



Amplitude Modulated Wave
(Including information signal guide)



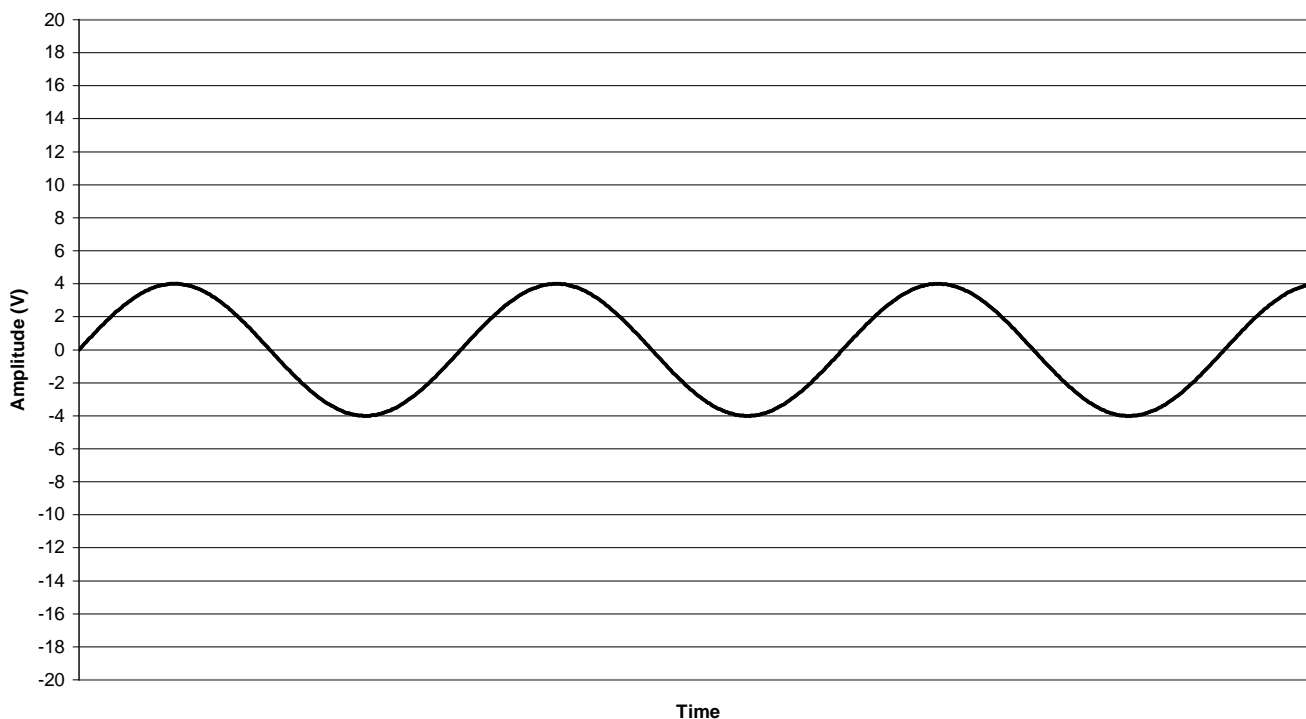
Topic 4.3.2 - Amplitude Modulation



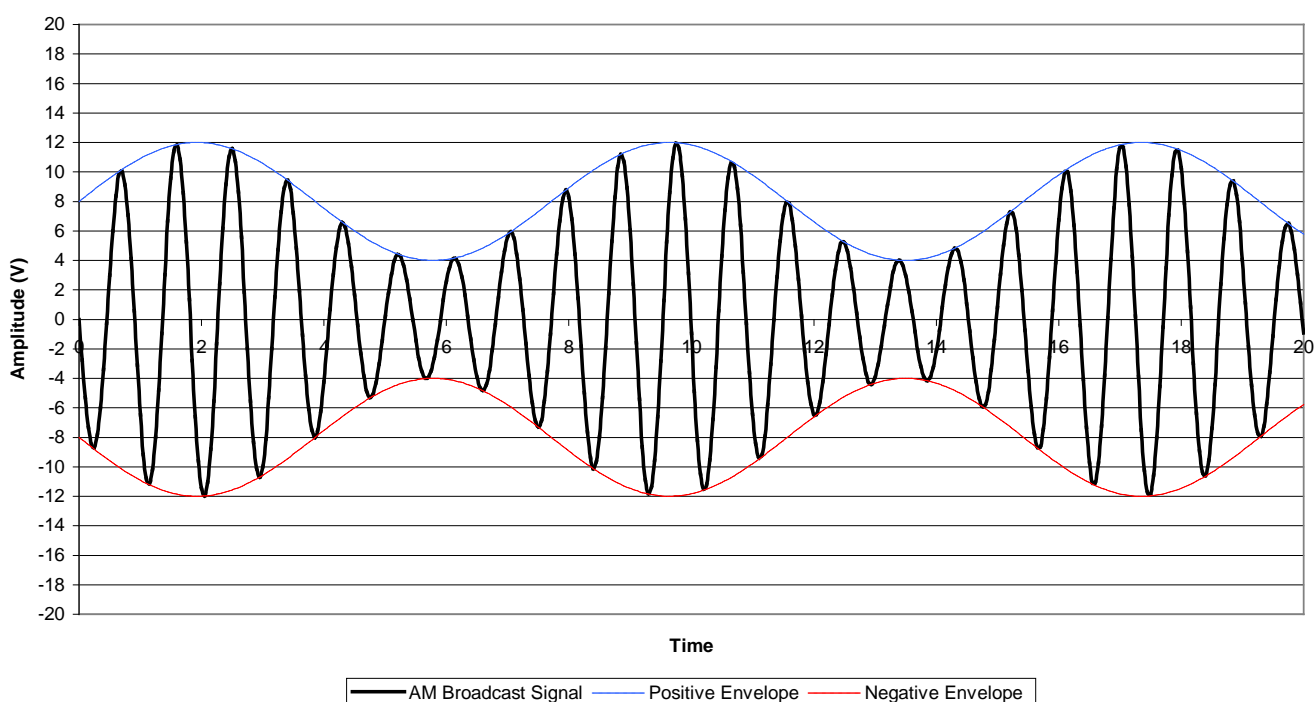
Example 2.

In this example, the information signal has an amplitude of 4V.

Information Signal Wave



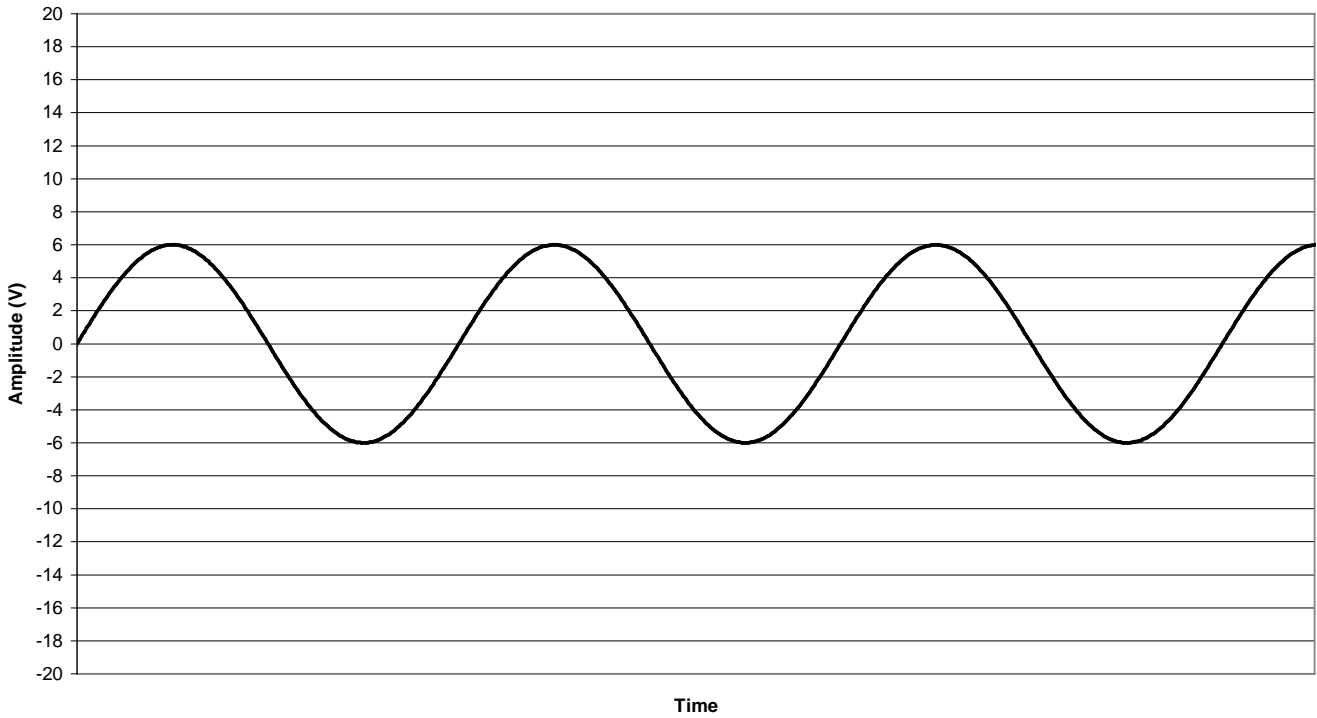
Amplitude Modulated Wave
(Including information signal guide)



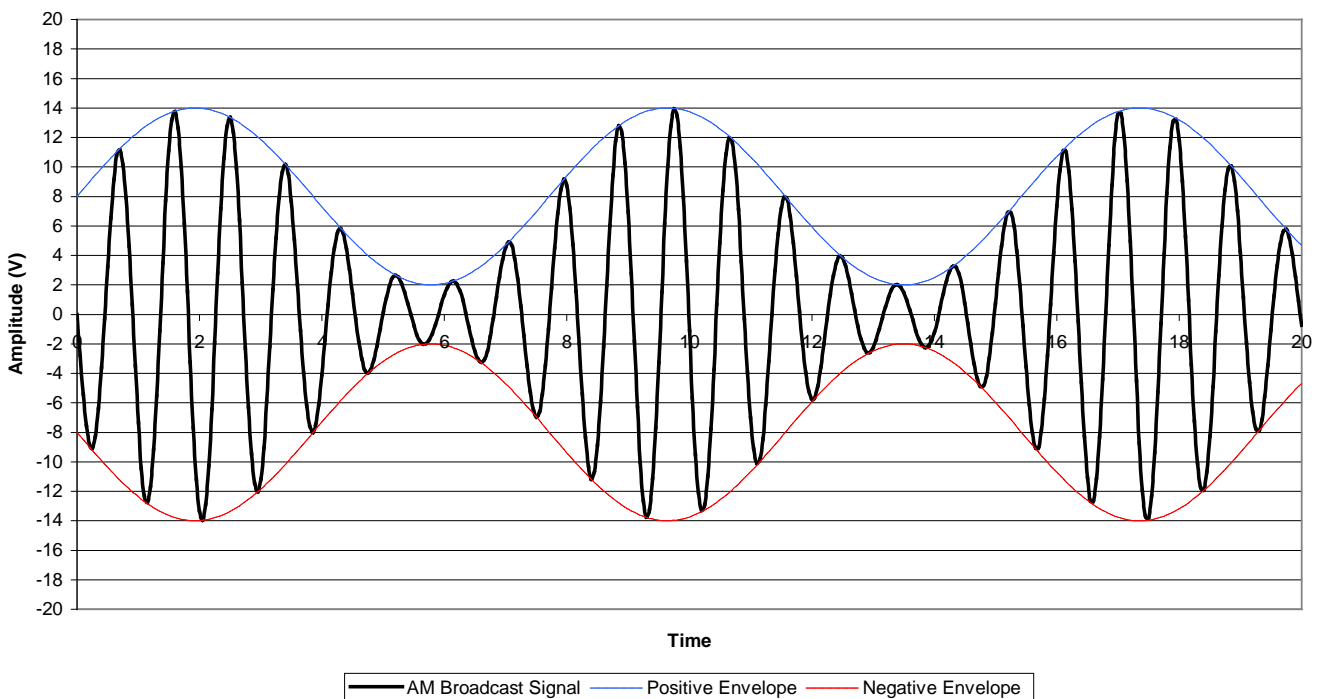
Example 3.

In this example, the information signal has an amplitude of 6V.

Information Signal Wave



Amplitude Modulated Wave
(Including information signal guide)



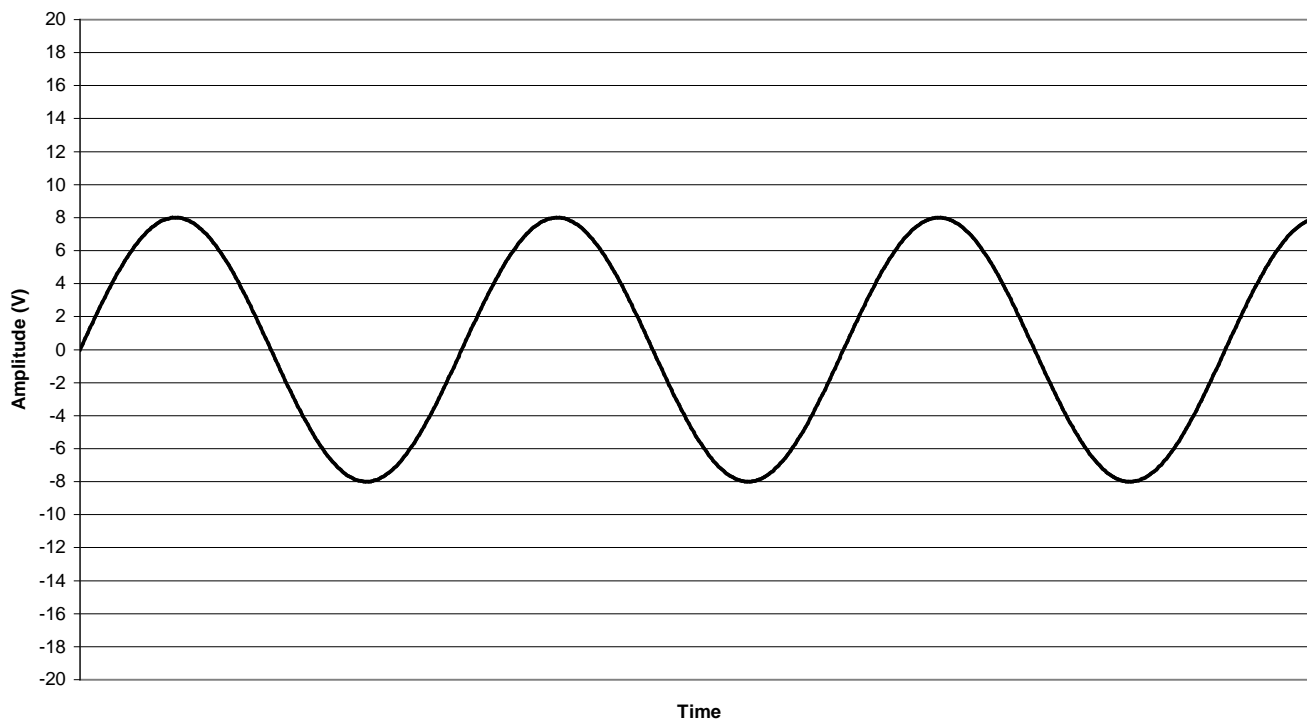
Topic 4.3.2 - Amplitude Modulation



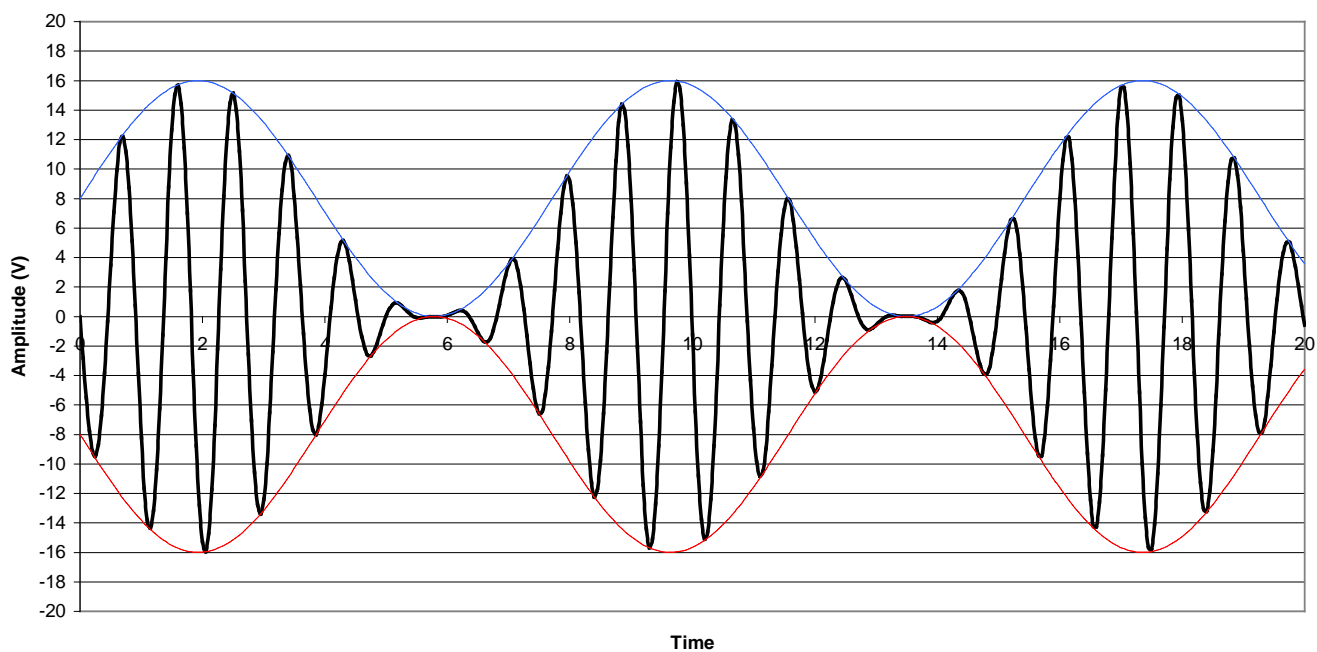
Example 4.

In this example, the information signal has an amplitude of 8V.

Information Signal Wave



Amplitude Modulated Wave
(Including information signal guide)

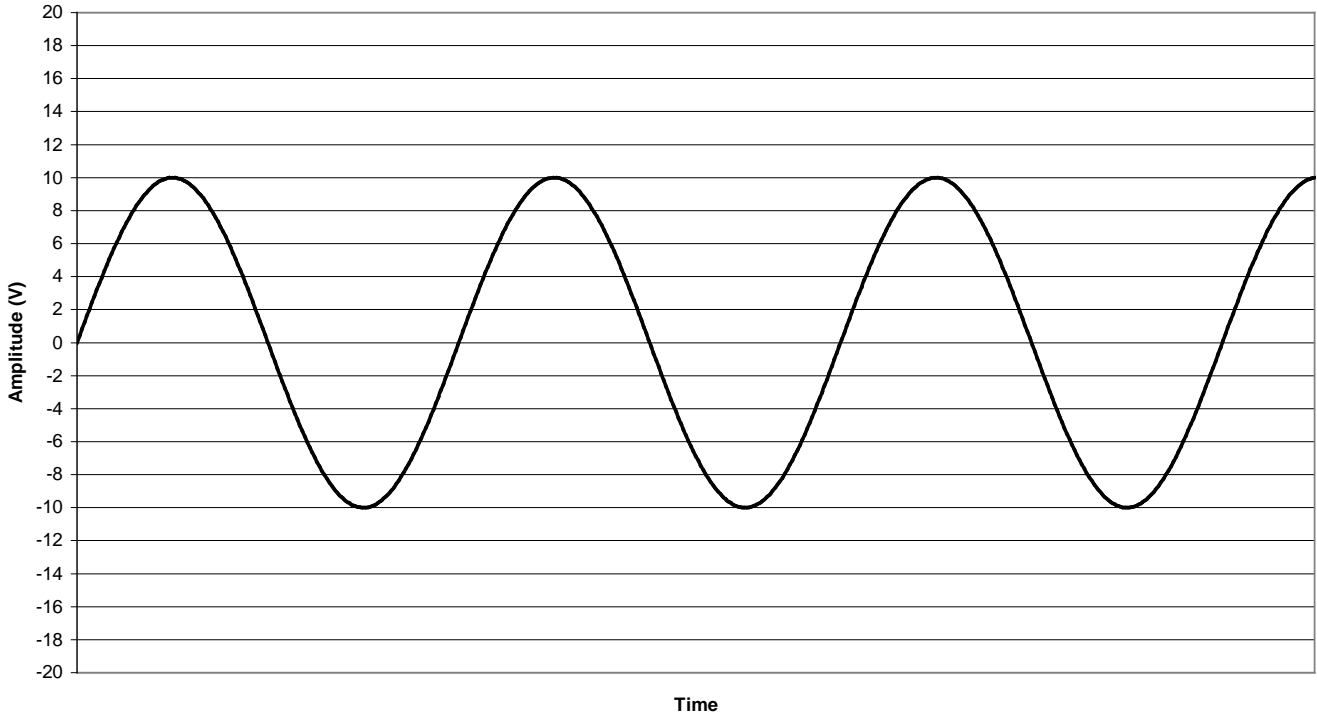


— AM Broadcast Signal — Positive Envelope — Negative Envelope

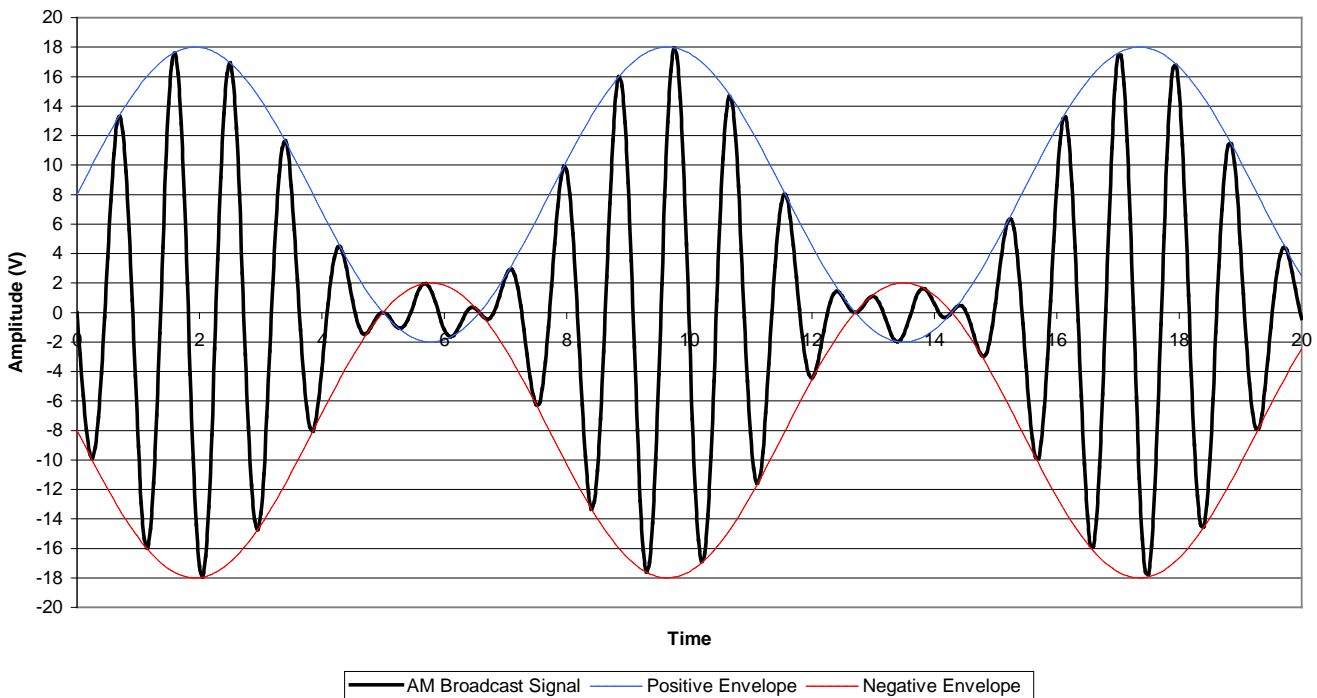
Example 5.

In this example, the information signal has an amplitude of 10V.

Information Signal Wave



Amplitude Modulated Wave
(Including information signal guide)



Topic 4.3.2 - Amplitude Modulation



The previous diagrams should have given you some indication of what happens when the information signal is amplitude modulated with a carrier of fixed amplitude. As the signal amplitude increases the carrier amplitude becomes significantly more varied.

When the signal amplitude and carrier amplitude are equal as in Example 4, there is a point where the carrier amplitude actually reaches 0V. In the case of Example 5 where the information signal amplitude is larger than that of the carrier, an amplitude modulated signal is produced, but there are areas where the information signal overlaps the 0V carrier amplitude and this would lead to distortion if it actually occurred during transmission. This variation in amplitude is known as the depth of modulation.

Being able to calculate the depth of modulation more accurately would be beneficial, and can be achieved simply by using the following equation. Depth of modulation is given the symbol m and the equation is

$$m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$$

V_{\max} is the maximum amplitude of the AM signal.

V_{\min} is the minimum amplitude of the AM signal.

If we consider Example 1, $V_{\max} = 10V$, $V_{\min} = 6V$. Therefore we can calculate the depth of modulation by using:

$$\begin{aligned} m &= \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\% \\ m &= \frac{(10 - 6)}{(10 + 6)} \times 100\% \\ m &= \frac{4}{16} \times 100\% = 25\% \end{aligned}$$

You can use a similar analysis for Examples 2, 3, 4, and 5, and it will be left to you to carry out this exercise in your own time. Worked solutions will be found on page 20 at the end of these notes - just in case you need them.

Using this formula is all well and good provided that you are given a graph or AM wave to work with, but what if you only have the amplitude of the carrier and the information signal ?

All is not lost, so don't worry. There is another version of the depth of modulation formula which deals only with the amplitudes of the carrier and information signals. The formula is as follows:

$$m = \frac{A_s}{A_c} \times 100\%$$

Where A_s = information signal amplitude, and
 A_c = carrier signal amplitude

Let us consider Example 1 again, now looking at the amplitudes of the signal and carrier only. $A_s = 2V$, $A_c = 8V$, therefore depth of modulation is given by:

$$m = \frac{A_s}{A_c} \times 100\%$$
$$m = \frac{2}{8} \times 100\% = 25\%$$

So now it does not matter whether we are using an amplitude modulated carrier or dealing with an individual carrier and information signal we will always be able to determine the depth of modulation.

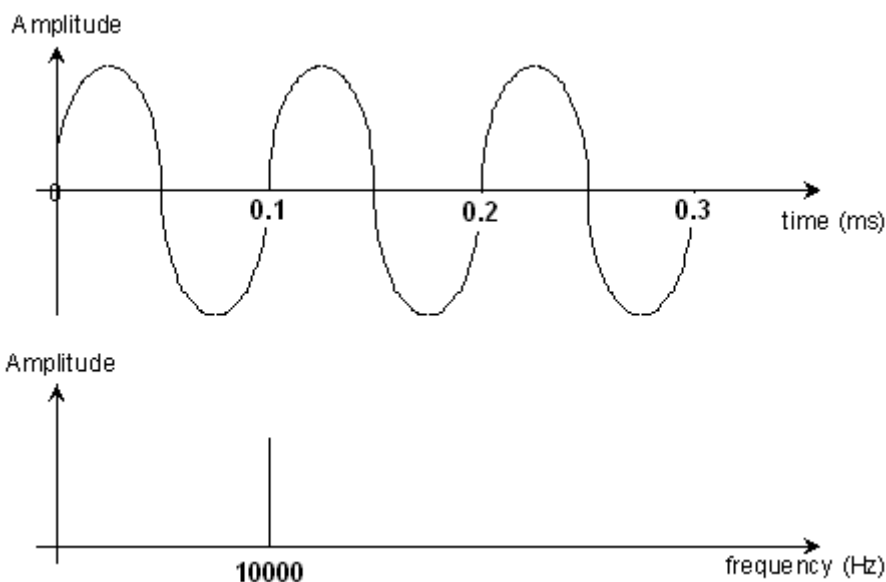
Remember that anything over 100% modulation will result in distortion of the received signal, and that in practice the depth of modulation does not usually exceed 80%, to provide a safety zone and maintain signal clarity.

The only remaining thing to look at with AM signals is their frequency spectrum analysis. In Topic 4.2.1. we discussed the frequency spectrum of a pure tone and a complex wave, e.g. a square wave. We will now look briefly at what happens to the frequency spectrum of an AM wave.

Topic 4.3.2 - Amplitude Modulation



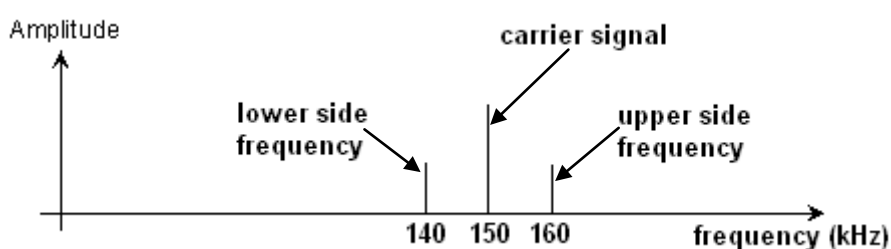
If we start with the simplest of all signals which will be a pure tone of fixed frequency. Let us assume that the pure tone to be transmitted is at a frequency of 10kHz. The signal and its spectrum will be as follows:



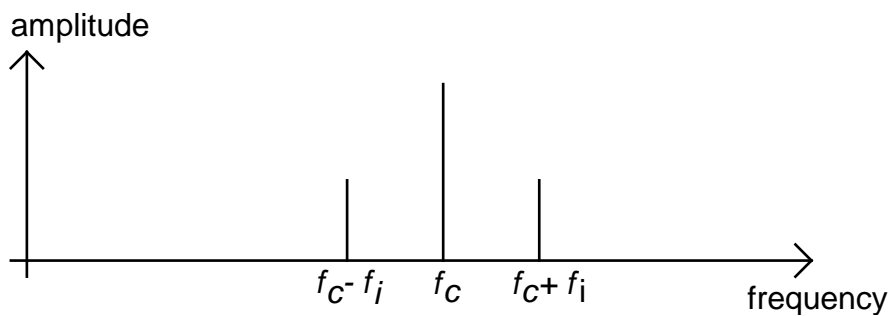
This information signal is to be transmitted using a carrier frequency of 150kHz. This will also be a pure tone of frequency 150kHz. The spectrum of this carrier signal will be as follows:



When the two signals, i.e. carrier and information are combined using Amplitude Modulation, from page 4, the resulting signal contains three components, (i) the original carrier frequency f_c and original amplitude A_c (ii) a wave of frequency $f_c - f_i$, called the lower side frequency and (iii) a wave of frequency $f_c + f_i$, called the upper side frequency. The resulting spectrum is:



In general, the following diagram shows the spectrum of an AM signal composed of a sinusoidal carrier, amplitude modulated by a pure sinusoidal information signal. Remember that a spectrum diagram plots amplitude against frequency.



In a real communication system, the information signal is highly unlikely to be a pure sine wave which has constant frequency and amplitude.

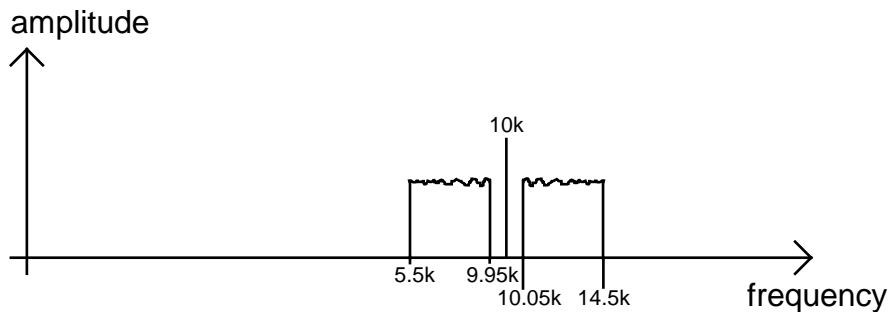
Real signals will be complex waves made up of a range of frequencies as discussed in the work on filters earlier. This range of frequencies will make up the signal **baseband**.

Each frequency in this baseband is likely to have a range of amplitudes that will be constantly changing, and any attempt to draw them would result in a very fuzzy picture. When presented with a real signal, for example a low quality music signal containing frequencies in the range 50 - 4500 Hz, this would be represented by the following spectrum diagram.

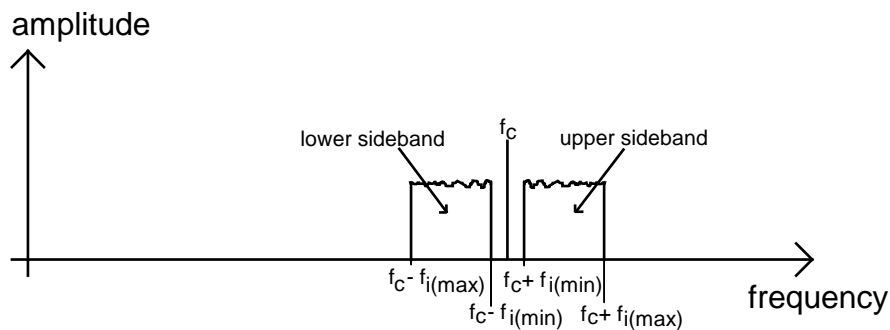


If this information signal is then used to amplitude modulate a sinusoidal carrier signal of 10kHz then the resulting spectrum would be as follows.

Topic 4.3.2 - Amplitude Modulation

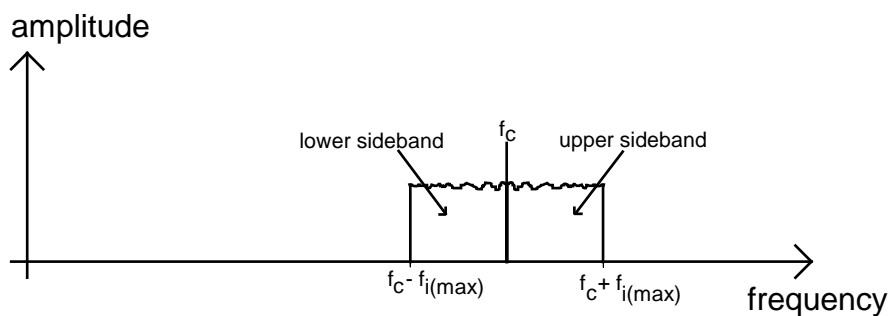


In general if a sinusoidal carrier f_c is amplitude modulated by an information baseband from $f_{i(min)}$ to $f_{i(max)}$, then the spectrum will be as shown below.



In general the lower frequency of the baseband is so small compared to the carrier frequency, that the diagram is difficult to draw accurately because the gap between the carrier frequency and the start of the sidebands is so small that it is hardly noticeable.

It is common practice therefore to draw the spectrum as follows.



The bandwidth required for an AM waveform is twice the maximum frequency in the information signal.

It should also be observed that:

- The spectrum is composed of the carrier and the upper and lower sidebands;
- The carrier component is unchanged by the modulation process.
- The information is in the sidebands and not in the carrier.
- The information is duplicated in the two sidebands.

Again for the enthusiast!

The resulting signal from this form of Amplitude Modulation is known as **Double-SideBand Amplitude Modulation (DSB-AM)**. Surprisingly this process is not very efficient. At best when the carrier is subject to 100% modulation i.e. $A_i = A_c$, half the transmitted power is in the sinusoidal carrier, which contains no information, and the other half is shared between the two sidebands, which contain the same information. So at best, the part of the signal containing the information has just one quarter of the transmitted power.

The process can be made more efficient by eliminating the carrier and transmitting only the two sidebands. This would be called **Double SideBand Suppressed Carrier (DSB-SC)**. The bandwidth required for this type of transmission will not change however, and so has limited advantage for the extra work needed to remove the carrier.

The most efficient form of AM transmission is achieved when the carrier and one of the sideband frequencies are filtered out of the transmission, This is called **Single SideBand Suppressed Carrier (SSB-SC)**. This form of transmission is not only more efficient but it also halves the bandwidth requirement.

It might be expected that this was the form of AM in use today because of the saving in efficiency and bandwidth. However this is not the case, although it is more efficient, it is also the most problematic to demodulate.

It is therefore necessary to have more complex receivers which inevitably make them more expensive and less attractive for people to buy, particularly when the quality of the received signal is not improved.

Examples of Information transmitted using AM.

Radio:

Ever since radio became commercially viable DSB-AM has been used on the long-wave and medium-wave bands. This is because the receivers for this type of transmission are relatively uncomplicated, making them affordable for the majority of the population who have been encouraged to buy and use them.

Television:

The brightness or luminance information signal that creates the picture on the television screen for a normal terrestrial station (e.g. BBC1, BBC2, ITV1 etc rather than the 'new' digital services like BBC3 or Sky) is transmitted using a carrier with one full sideband and one small part of the other sideband. The small part is called a '*vestige*'. The transmission process using this technique is called **Vestigial Sideband Amplitude Modulation**. The purpose of using such a technique is to reduce the bandwidth required to broadcast the signal.

Mobile Phones:

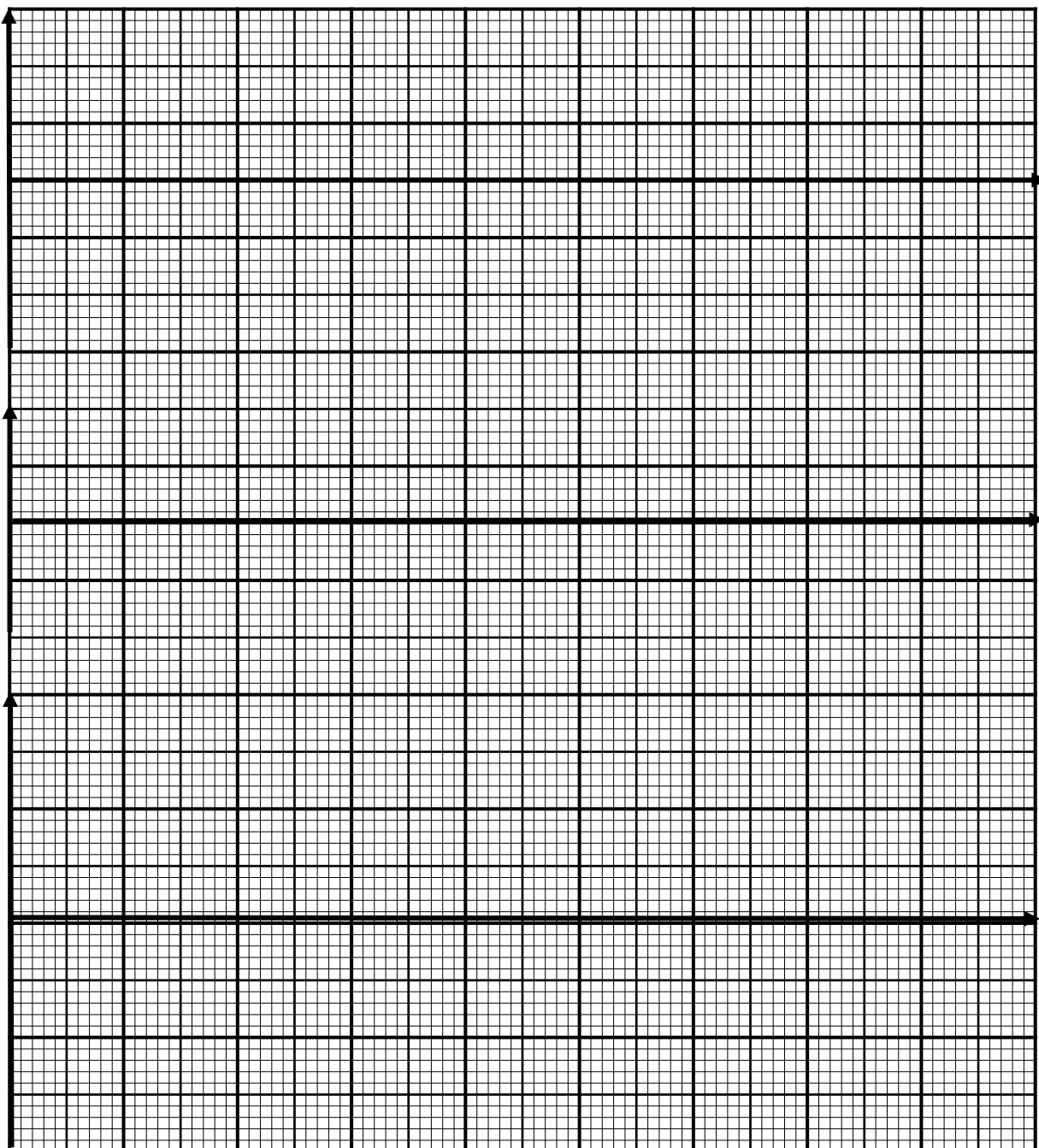
Some of the early mobile phones made use of single sideband AM to maximise the use of the available bandwidth. This however made the receivers more expensive, and since moving to the microwave carrier frequency range where bandwidth has become less of an issue, SSB-AM has been abandoned to enable cheaper demodulation techniques to be employed.

Radio Amateurs:

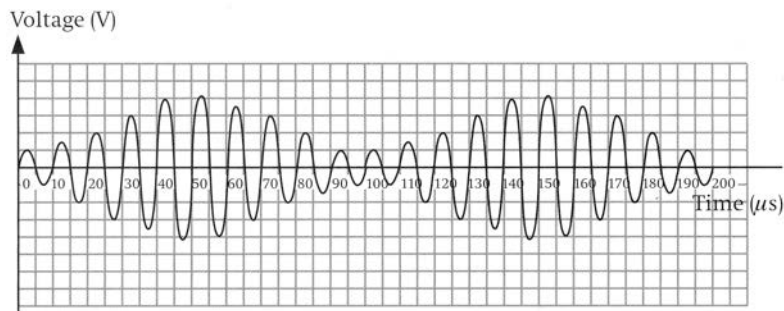
Radio Amateurs or Radio Hams as they are often called, have always used Single Sideband Transmissions, specifically because this reduces the bandwidth requirement so more transmissions can be setup in the available bandwidth. It also has the interesting property that when the information signal drops to zero (i.e. during a gap in conversation, or silences) there is no transmitted power at all.

Student Exercise 1:

1. A 100kHz carrier is amplitude modulated by a 10 kHz square wave. The carrier has an amplitude of 20V and the square wave has an amplitude of 5V. Use the axes on the graph paper below to draw (i) 15 cycles of the carrier wave on the first set of axes, (ii) the 10kHz square wave on the second set of axes using the same time scale, (iii) the resulting AM signal, labelling important points on the voltage and time axes.



2. The diagram below shows an AM carrier modulated by a pure tone.



(i) Calculate the carrier frequency and the pure tone frequency.

Carrier Frequency =

Pure Tone Frequency =

(ii) Sketch the frequency spectrum of the AM signal.



Topic 4.3.2 - Amplitude Modulation



3. A radio station on the MW network transmits with a carrier frequency of 500 kHz, amplitude modulated by an audio baseband of 100 Hz to 3.5 kHz. Sketch the frequency spectrum of the station and calculate the bandwidth of the AM signal.



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Solutions to Depth of Modulation Task.

Example 2, $V_{\max} = 12V$, $V_{\min} = 4V$.

$$m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$$

$$m = \frac{(12 - 4)}{(12 + 4)} \times 100\%$$

$$m = \frac{8}{16} \times 100\% = 50\%$$

Example 3, $V_{\max} = 14V$, $V_{\min} = 2V$.

$$m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$$

$$m = \frac{(14 - 2)}{(14 + 2)} \times 100\%$$

$$m = \frac{12}{16} \times 100\% = 75\%$$

Example 4, $V_{\max} = 16V$, $V_{\min} = 0V$.

$$m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$$

$$m = \frac{(16 - 0)}{(16 + 0)} \times 100\%$$

$$m = \frac{16}{16} \times 100\% = 100\%$$

Example 5, $V_{\max} = 18V$, $V_{\min} = -2V$.

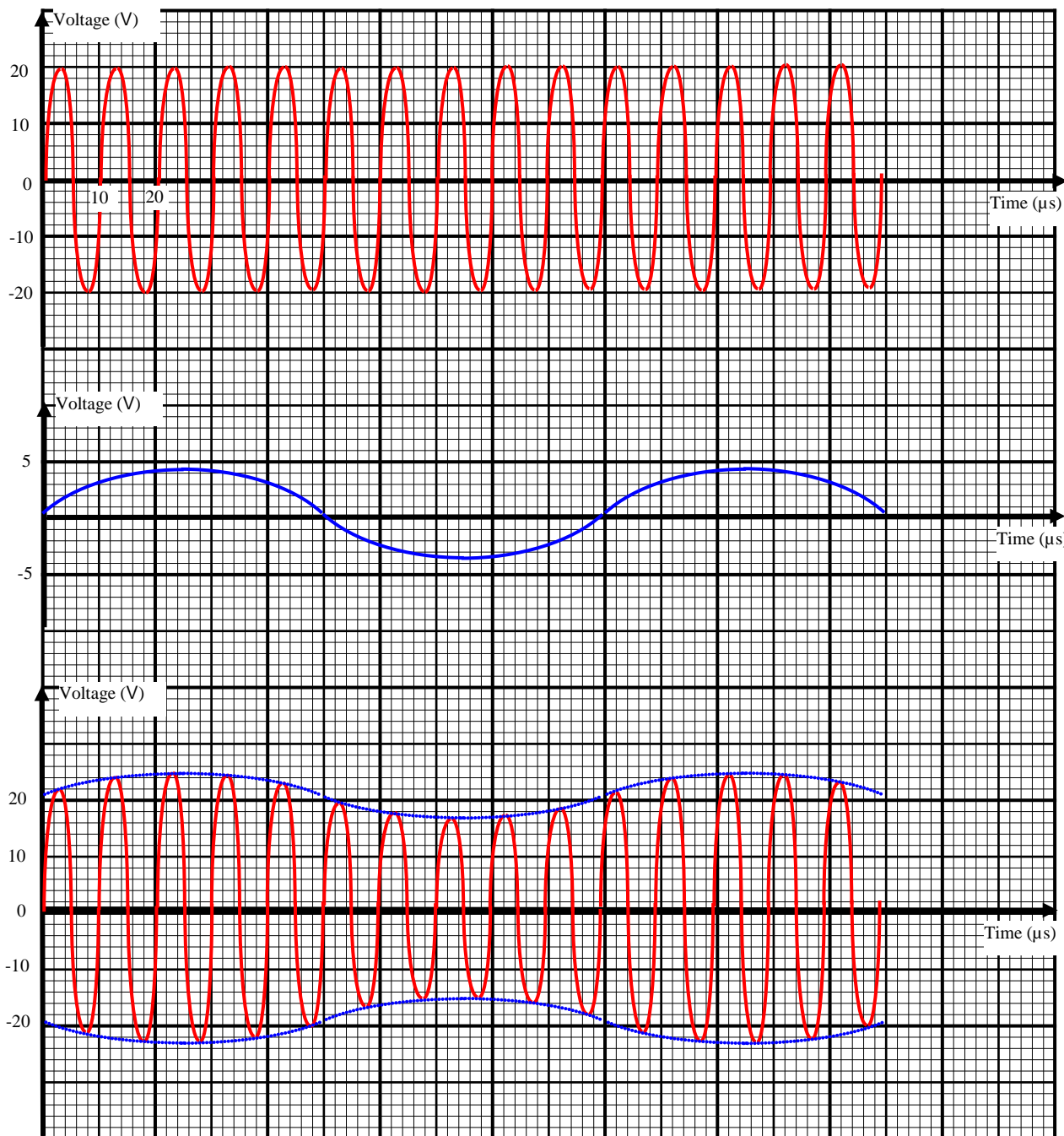
$$m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$$

$$m = \frac{(18 - (-2))}{(18 + (-2))} \times 100\%$$

$$m = \frac{20}{16} \times 100\% = 125\%$$

Solutions to Student Exercise 1:

1.



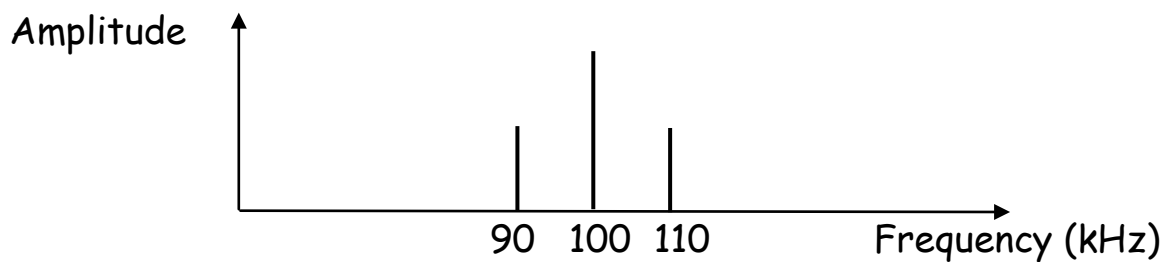
2. (i) Carrier frequency: 1 cycle lasts $10\mu s$

$$\text{Therefore } f_c = \frac{1}{10\mu s} = 100\text{kHz}$$

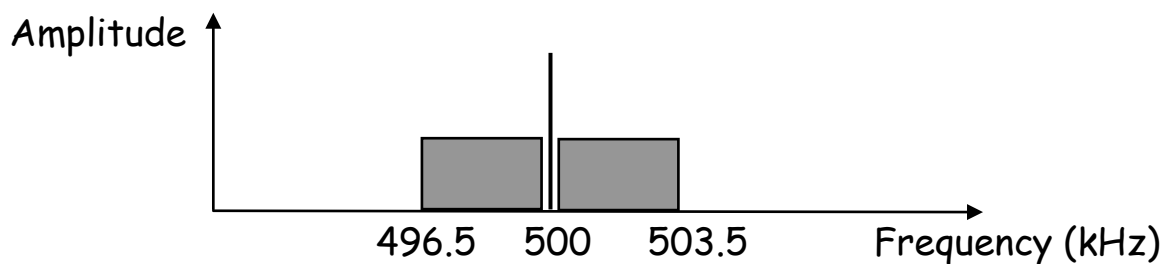
Signal frequency : 1 cycle from peak to peak = $150\mu s - 50\mu s = 100\mu s$.

$$\text{Therefore } f_s = \frac{1}{100\mu s} = 10\text{kHz}$$

(ii) Sketch the frequency spectrum of the AM signal.



3.



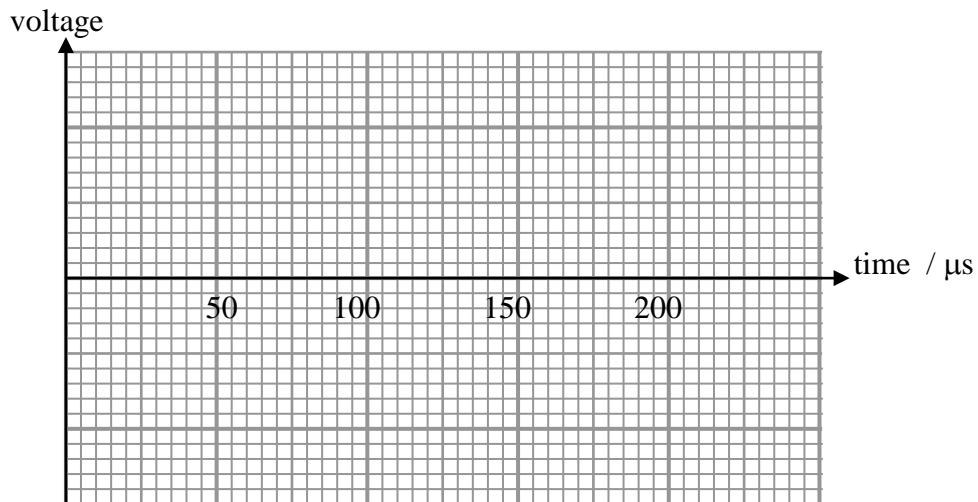
$$\text{Bandwidth} = 503.5 - 496.5 = 7\text{kHz}$$

Now for some past exam questions for additional practice:

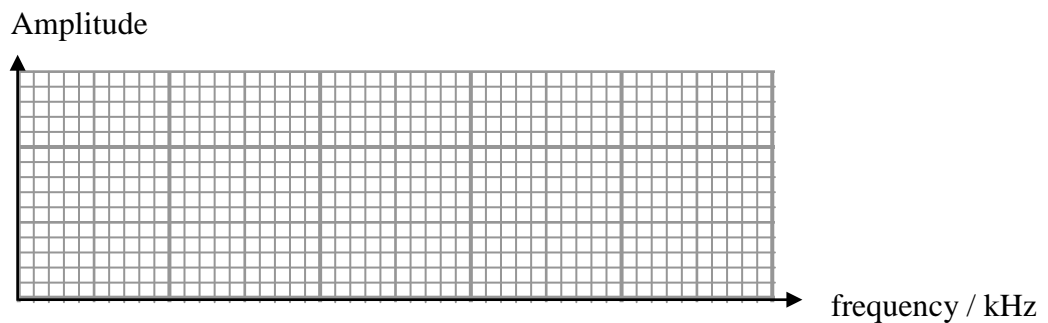
Topic 4.3.2 - Amplitude Modulation



1. A radio station broadcasts for 20 hours per day on a frequency of 100 kHz, using Amplitude Modulation. During the **4 hours** when the radio station is 'off-air' the station transmits its call-sign, a continuous 10kHz single frequency with a modulation depth of 50%.
- a. Use the axes provided below to sketch the amplitude modulated envelope of the carrier wave.

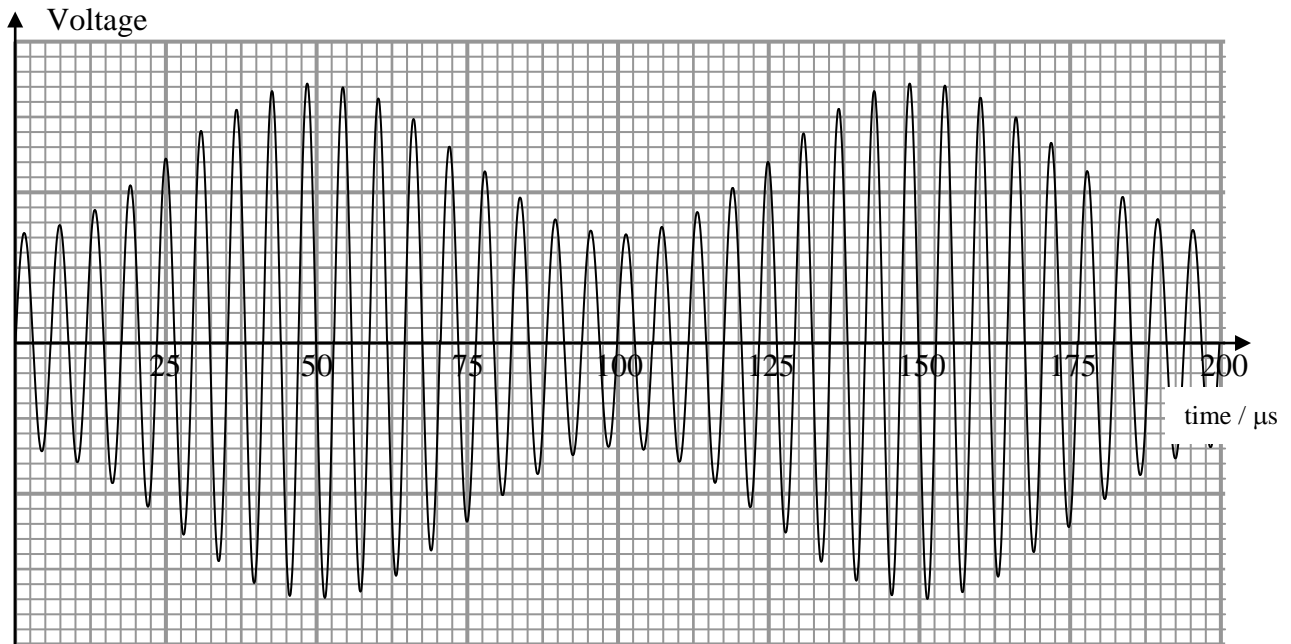


- b. Using the axes below sketch the frequency spectrum diagram for the *amplitude modulated carrier* of the radio station when transmitting its call-sign. Label all frequencies. [3]



[3]

2. A carrier wave is amplitude modulated by a single frequency test signal and broadcast by a radio station. An engineer received the following signal on the receiving circuit.



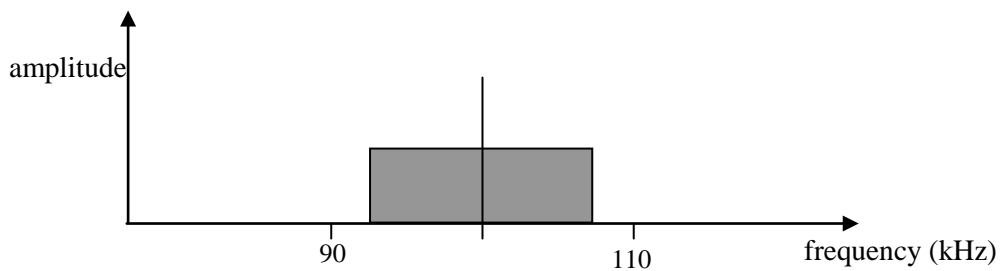
- a. Determine the period and frequency of the test signal.

Period :

Frequency :

[2]

- b. The single frequency test signal is now replaced by an audio signal. The frequency spectrum of this broadcast is shown below.



- i. What is the carrier frequency of the radio station ?

.....

[1]

- ii. What is the broadcast bandwidth of this transmission ?

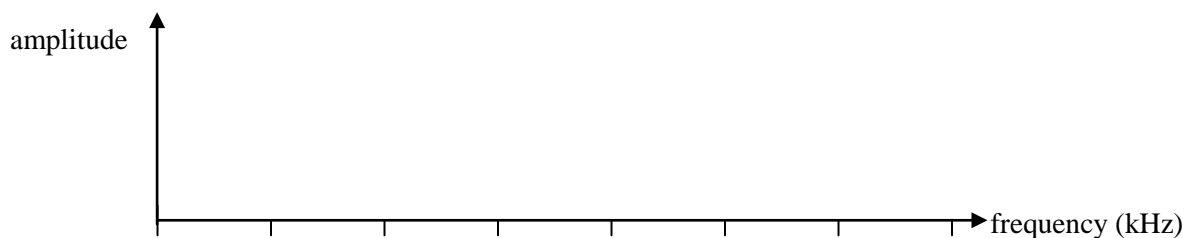
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[1]

Topic 4.3.2 - Amplitude Modulation

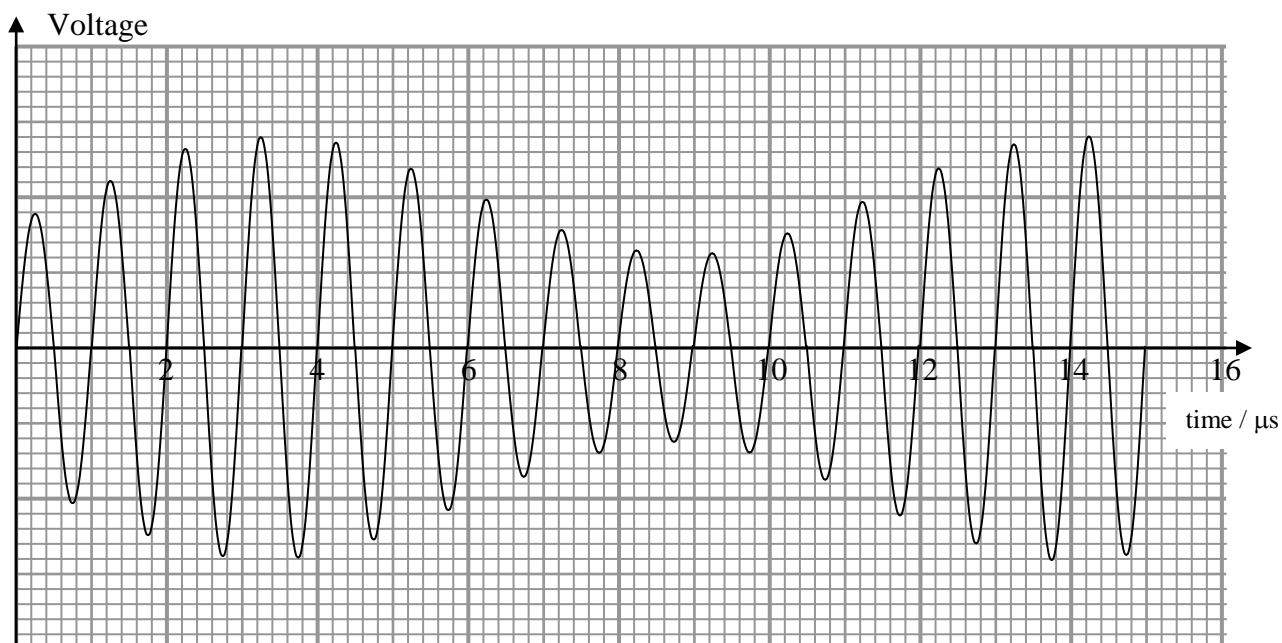


- iii. Use the axes provided to sketch the frequency spectrum for the **audio signal** being transmitted. Label the frequency axis with the appropriate scale.



[2]

3. In a radio broadcast, the audio signal from a microphone is amplitude modulated onto a carrier wave.



- (a) The graph shows part of the amplitude modulated signal.
- (i) What are the period and frequency of the audio signal in the graph ?

Period :

Frequency :

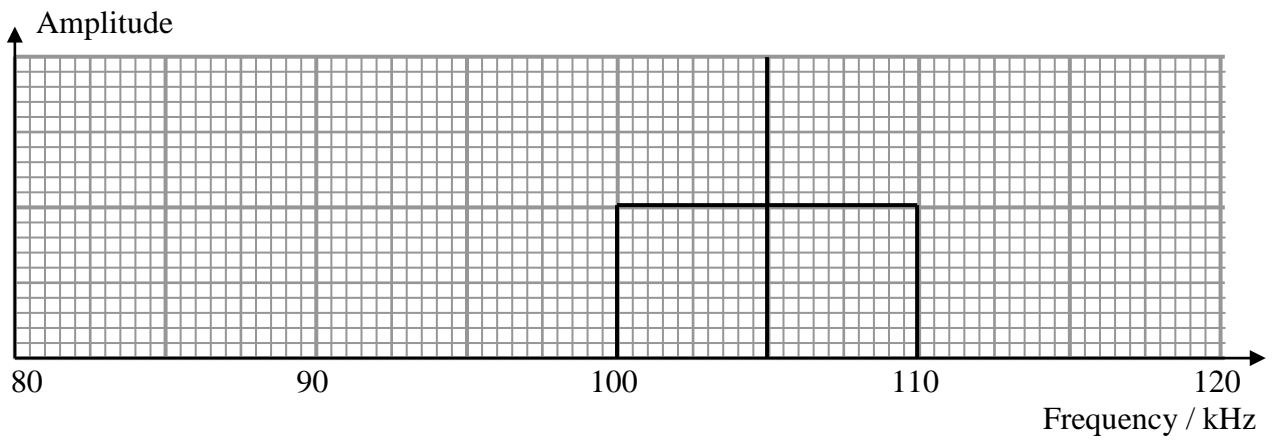
[2]

- (ii) Estimate the depth of modulation of the carrier wave.

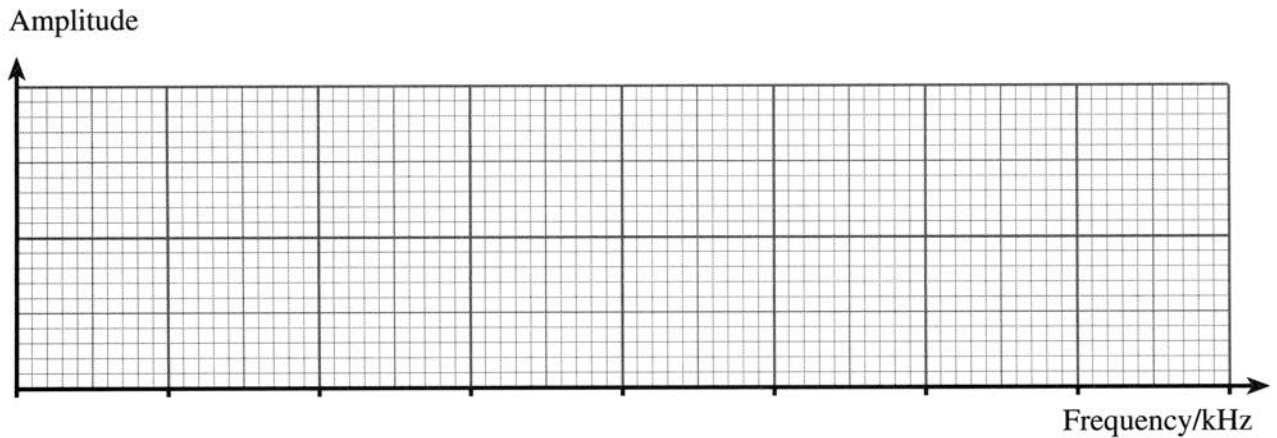
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[1]




(b) The frequency spectrum of a later broadcast is shown below.



- (i) What is the carrier frequency of the radio station?
..... [1]
- (ii) What is the broadcast bandwidth of this transmission ?
..... [1]
- (iii) Use the axes provided to sketch the frequency spectrum for the **audio signal** being transmitted. [2]



Self Evaluation Review

Learning Objectives	My personal review of these objectives:		
			
sketch and recognise the resulting waveforms for a sinusoidal carrier being amplitude modulated by a single frequency audio signal;			
draw and analyse graphs to show the resulting waveform, and frequency spectrum for a sinusoidal carrier amplitude modulated by an audio signal, to a given depth of modulation, m;			
select and use the formula: $m = \frac{(V_{\max} - V_{\min})}{(V_{\max} + V_{\min})} \times 100\%$ to calculate the depth of modulation for a given amplitude modulated RF signal.			

- Targets: 1.
-
2.
-