Topic 4.3.3 – Frequency Modulation

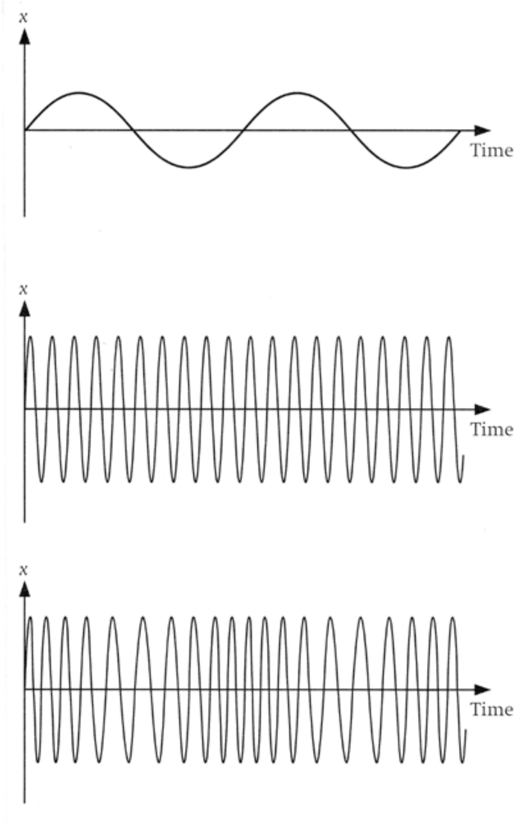
Learning Objectives:

At the end of this topic you will be able to;

- ☑ sketch, recognise and analyse the resulting waveforms for a sinusoidal carrier being frequency modulated by a single frequency audio signal;
- ☑ recall that an FM-modulated carrier produces an infinite number of sidebands;
- ☑ recall that frequency deviation Δf_c is the maximum change in frequency of the carrier from its base value f_c;
- ☑ recall that the modulation index β is the FM equivalent to the depth of modulation;
- ☑ use the formula $\beta = \frac{\Delta f_c}{f_i}$, where f_i is the maximum frequency of the modulation signal;
- ☑ appreciate that almost all the power of a transmitted FM signal is contained within a bandwidth of $2(1+\beta) f_i$, where f_i is the maximum frequency of the modulating signal;
- ☑ recognise the frequency spectrum diagram for a sinusoidal carrier being frequency modulated by a single audio signal for $\beta < 1$, $\beta = 1$ and $\beta = 3$.

Frequency Modulation

In **F**requency **M**odulation (**FM**) the instantaneous value of the information signal controls the frequency of the carrier wave. This is illustrated in the following diagrams.



Notice that as the information signal increases, the frequency of the carrier increases, and as the information signal decreases, the frequency of the carrier decreases.

The frequency f_i of the information signal controls the rate at which the carrier frequency increases and decreases. As with AM, f_i must be less than f_c. The amplitude of the carrier remains constant throughout this process.

When the information voltage reaches its maximum value then the change in frequency of the carrier will have also reached its maximum deviation above the nominal value. Similarly when the information reaches a minimum the carrier will be at its lowest frequency below the nominal carrier frequency value. When the information signal is zero, then no deviation of the carrier will occur.

Topic 4.3.3 – Frequency Modulation

WJEC CBAC

The maximum change that can occur to the carrier from its base value f_c is called the **frequency deviation**, and is given the symbol Δf_c. This sets the *dynamic range* (i.e. voltage range) of the transmission.

The *dynamic range* is the ratio of the largest and smallest analogue information signals that can be transmitted.

Worked Example:

A 400kHz sinusoidal carrier of amplitude 5V is frequency modulated by a 3kHz sinusoidal information signal of amplitude 3V. The behaviour of the carrier is governed by the frequency deviation per volt and for this system is 25kHz per volt. Describe how the resulting FM signal changes with time.

Solution:
The FM carrier will change in frequency from 400 kHz *to 475* kHz *to 400* kHz *to 325* kHz *and back to 400* kHz*, 3000 times per second. This is because the frequency deviation* Δf_c *= 3 x 25* kHz *= 75* kHz. *The amplitude of the carrier will remain fixed at 5* V.

If the same system was used and the amplitude of the information signal was decreased to 1V, how would this affect the resulting FM signal? Describe the changes in the space below, including any relevant calculations.

....

....

....

....

....

....

Modulation Index

All FM transmissions are governed by a modulation index, β, which controls the dynamic range of the information being carried in the transmission. The modulation index, β, is the ratio of the frequency deviation, Δf_c, to the maximum information frequency, f_i, as shown below:

$$\beta = \frac{\Delta f_c}{f_i}$$

<u>For the Enthusiast !</u>

The exact way in which the frequency modulated carrier is produced is very complex and involves very advanced mathematics. A summary of the solution is provided here for those who are enthusiastic about such things. The general mathematical formula for a sinusoidal wave is :

$$V = V_{\max} \sin 2\pi f t$$

Where V = instantaneous value of voltage, V_{max} = maximum amplitude of the wave, f = frequency of wave and t = time.

For a carrier wave having a frequency f_c and amplitude A_c, the instantaneous value V_c can be obtained using the following equation.

$$V_c = A_c \sin 2\pi f_c t$$

The simplest information signal that can be applied will be another pure sine wave. Assume that this has a frequency f_i and amplitude A_i, the instantaneous value V_i can be obtained using the following equation.

$$V_i = A_i \sin 2\pi f_i t$$

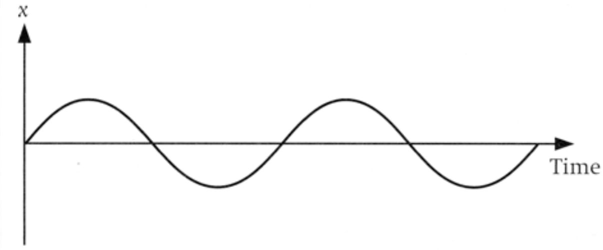
When the carrier is frequency modulated, the resulting wave is governed by the equation:

$$V_{FM} = A_c \sin(2\pi f_c t + \beta \sin 2\pi f_i t)$$

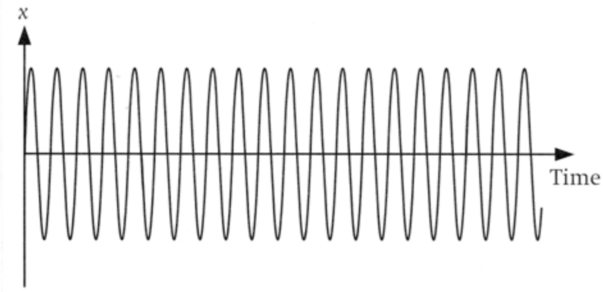
Where β is known as the modulation index, and defined by the equation: $\beta = \dfrac{\Delta f_c}{f_i}$

Topic 4.3.3 – Frequency Modulation

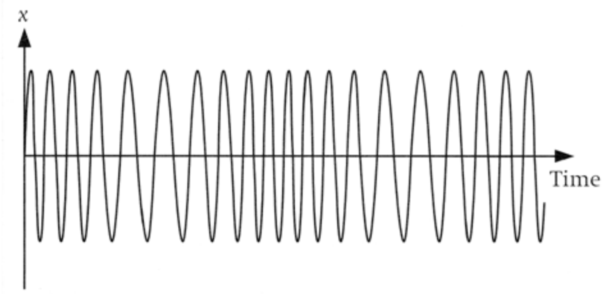
The diagrams opposite show examples of how the modulation index affects the FM output, for a simple sinusoidal information signal of fixed frequency. The carrier signal has a frequency of ten times that of the information signal.



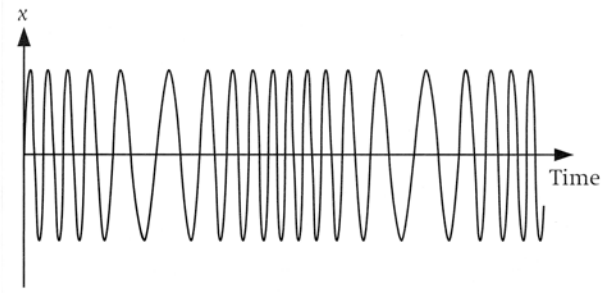
The first graph shows the information signal, the second shows the unmodulated carrier.



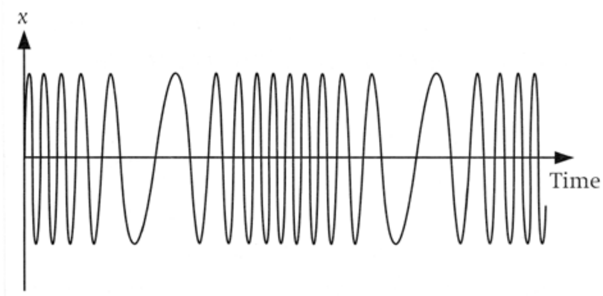
This graph shows the frequency modulated carrier when the modulation index = 3.



This graph shows the frequency modulated carrier when the modulation index = 5.



This graph shows the frequency modulated carrier when the modulation index = 7.



As the modulation index increases you should notice that the peaks of the high frequency get closer together and low frequency get further apart. For the same information signal therefore, the carrier signal has a higher maximum frequency.

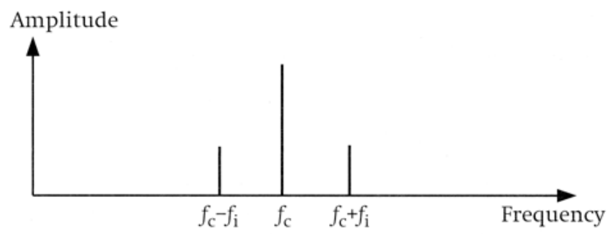
FM Spectra

When the amplitude of the frequency components of this simple FM waveform are plotted as a function of frequency, the resulting spectrum is much more complicated than that of the simple AM waveform (i.e. one carrier and two sidebands) discussed in the previous unit. This is because there are now multiple frequencies present in the FM signal, even for the transfer of a simple sinusoidal information signal.

The detailed analysis of an FM waveform is very complicated and well beyond the scope of this introductory course, however we will try to simplify things a little so that you get a flavour of what the key differences are.

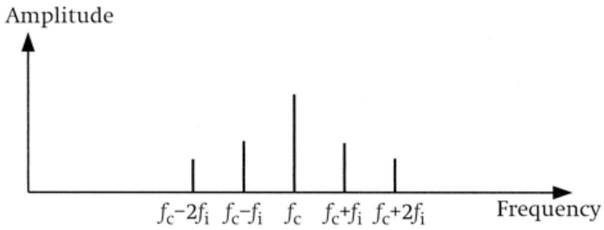
Theoretically, an FM spectrum has an infinite number of sidebands, spaced at multiples of f_i above and below the carrier frequency f_c . However the size and significance of these sidebands is very dependent on the modulation index, β . (As a general rule, any sidebands below 1% of the carrier can be ignored.)

If $\beta < 1$, then the spectrum looks like this:



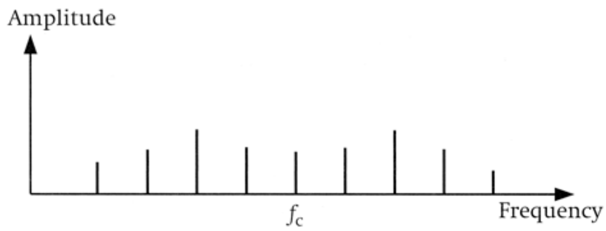
From the spectrum above it can be seen that there are only two significant sidebands, and thus the spectrum looks very similar to that for an AM carrier.

If $\beta = 1$, then the spectrum looks like this:



From the spectrum above we can see that the number of significant sidebands has increased to four.

If $\beta = 3$, then the spectrum looks like this:



From the spectrum above we can see that the number of significant sidebands has increased to eight.

It can be deduced that the number of significant sidebands in an FM transmission is given by $2(\beta + 1)$.

The implication for the bandwidth of an FM signal should now be coming clear. The practical bandwidth is going to be given by the number of significant sidebands multiplied by the width of each sideband (i.e. f_i).

$$
\begin{aligned}
Bandwidth_{FM} &= 2(\beta + 1) f_i \\
&= 2\left(\frac{\Delta f_c}{f_i} + 1\right) f_i \\
&= 2(\Delta f_c + f_i)
\end{aligned}
$$

The bandwidth of an FM waveform is therefore twice the sum of the frequency deviation and the maximum frequency in the information.

Additional Points to remember.

- An FM transmission is a constant power wave, regardless of the information signal or modulation index, β, because it is operated at a constant amplitude with symmetrical changes in frequency.
- As β increases, the relative amplitude of the carrier component decreases and may become much smaller than the amplitudes of the individual sidebands. The effect of this is that a much greater proportion of the transmitted power is in the sidebands (rather than in the carrier), which is more efficient than AM.

Determination of Bandwidth for FM Radio

FM radio uses a modulation index, β > 1, and this is called **wideband FM.** As its name suggests the bandwidth is much larger than AM.

In national radio broadcasts using FM, the frequency deviation of the carrier Δf_c, is chosen to be 75 kHz, and the information baseband is the high fidelity range 20 Hz to 15 kHz.

Thus the modulation index, β is 5 (i.e. 75 kHz $\div$ 15 kHz), and such a broadcast requires an FM signal bandwidth given by:

$$\begin{aligned} Bandwidth_{FM\ \ Radio} &= 2(\Delta f_c + f_{i(\max)}) \\ &= 2(75+15) \\ &= 180kHz \end{aligned}$$

For the enthusiast!

Further Examples of Information transmitted using F.M.

Mobile Phones:

Some mobile phone companies use FM with a very low modulation index, i.e. β < 1. This is known as **narrowband FM**. Mobile phone companies use this because it offers many of the advantages of FM, with the minimum bandwidth requirement.

Television Sound:

In terrestrial TV broadcasts, the video information is transmitted using AM as we saw during the previous section. This to make the most effective use of the bandwidth available. However the sound information is transmitted using FM, in order to reduce possible interference between the video and sound signals. In this case, the maximum deviation of the carrier, Δf_c , is chosen to be 50kHz, and the information baseband is again the high fidelity range 20Hz to 15kHz. Therefore the bandwidth required for TV Sound is:

$$Bandwidth_{TV\ \ Sound} = 2(\Delta f_c + f_{i(\max)})$$

$$= 2(50 + 15)$$

$$= 130kHz$$

Satellite TV.

Some satellite TV transmissions broadcast an analogue video signal using FM. This helps to obtain an acceptable signal at the receiving station even though the transmitter is some 36,000 km out into space! In this case, the maximum deviation of the carrier, Δf_c , is chosen to be about 10 MHz, with a video baseband of around 5MHz. Therefore the bandwidth required for SatelliteTV is:

$$Bandwidth_{Satellite\ \ TV} = 2(\Delta f_c + f_{i(\max)})$$

$$= 2(10 + 5)$$

$$= 30MHz$$

Note : An increasing number of satellite broadcasting companies are changing from analogue to digital formats. i.e. from Frequency Modulation to Pulse Code Modulation (Topic 4.3.5)

WJEC CBAC

Student Exercise 1:

1. A 10 MHz carrier is frequency modulated by a pure signal tone of frequency 8 kHz. The frequency deviation is 32 kHz. Calculate the bandwidth of the resulting FM waveform.

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

2. Suggest why it would not be sensible for long-wave radio transmitters operating in the range 140 kHz to 280 kHz to use FM.

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

3. An audio signal, with a base band of 200 Hz to 4 kHz, frequency modulates a carrier of frequency 50 MHz. The frequency deviation per volt is 10 kHz V^{-1} and the maximum audio voltage it can transmit is 3V. Calculate the frequency deviation and the bandwidth of the FM signal.

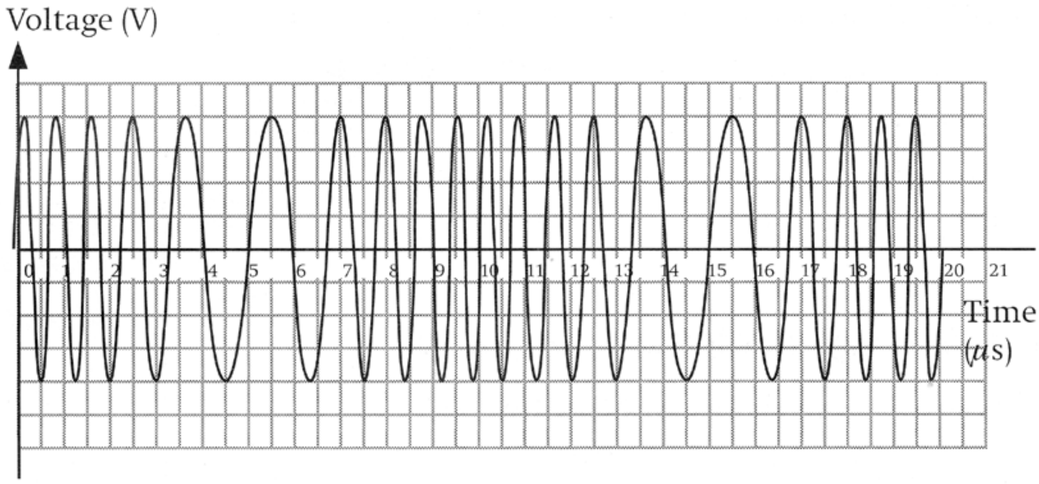
   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

   ……………………………………………………………………………………………………

4. The diagram below shows an FM carrier modulated by a pure tone (sinusoidal wave). Calculate the carrier frequency and the pure tone frequency.



....

....

....

....

Solutions to Student Exercise 1:

1.

$$\begin{aligned} Bandwidth &= 2(\Delta f_c + f_{i(\max)}) \\ &= 2(32+8) \\ &= 80kHz \end{aligned}$$

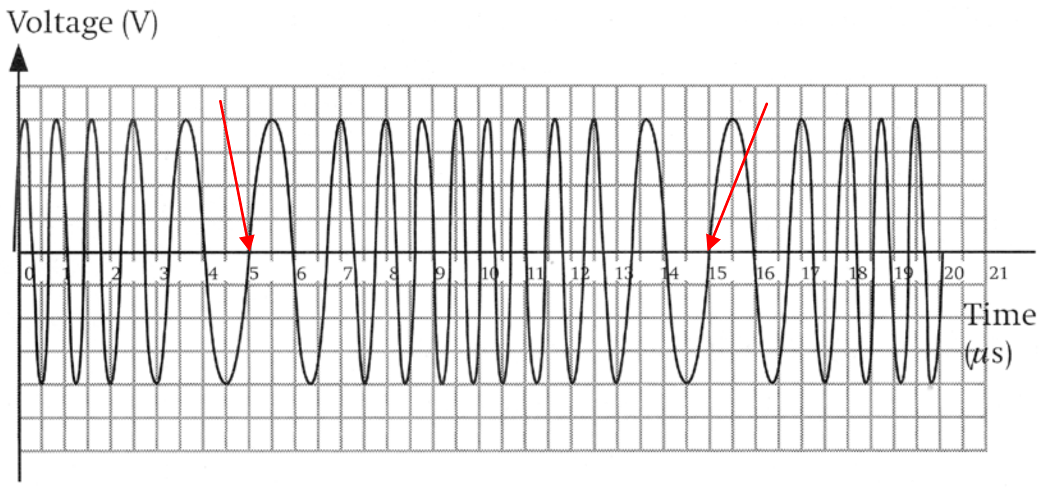
2. The long wave radio band is only 280 – 140 = 140 kHz wide. Typical broadcast bandwidths for FM transmissions are typically 180kHz, therefore the LW wave band is not big enough to accommodate an FM radio station.

3.

$$\begin{aligned} \Delta f_C &= 3V \times 10kHzV^{-1} \\ &= 30kHz \end{aligned}$$

$$\begin{aligned} Bandwidth &= 2(\Delta f_c + f_{i(\max)}) \\ &= 2(30+4) \\ &= 68kHz \end{aligned}$$

4.



Carrier frequency : Identify one cycle of the signal frequency, (most easily done by looking for the repeating widest waves as shown by the red arrows on the diagram above), then count the number of cycles that take place in this time. To get the time of one cycle just divide the time between the arrows by the number of complete cycles.

No of cycles = 10 Time = 15-5=10μs

Therefore time for 1 cycle = $\frac{10\mu s}{10} = 1\mu s$

Therefore $f_C = \frac{1}{T} = \frac{1}{1 \times 10^{-6}} = 1MHz$

Signal frequency : One cycle occurs between the two red arrows, equivalent to a time of 10μs.

Therefore $f_C = \frac{1}{T} = \frac{1}{10 \times 10^{-6}} = 100kHz$

Now for some examination-style questions.

Examination Style Questions.

1. A 24 MHz carrier is frequency modulated by a pure signal tone of frequency 12 kHz. The frequency deviation is 56 kHz. Calculate the bandwidth of the resulting FM waveform.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

[2]

2. Suggest why it would not be very sensible for medium-wave radio transmitters operating in the range 600 kHz to 1600 kHz to use FM.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

[2]

3. An audio signal, with a base band of 200 Hz to 12 kHz, frequency modulates a carrier of frequency 50 MHz. The frequency deviation per volt is 15 kHz V^{-1} and the maximum audio voltage it can transmit is 7 V. Calculate the frequency deviation and the bandwidth of the FM signal.

……………………………………………………………………………………………………

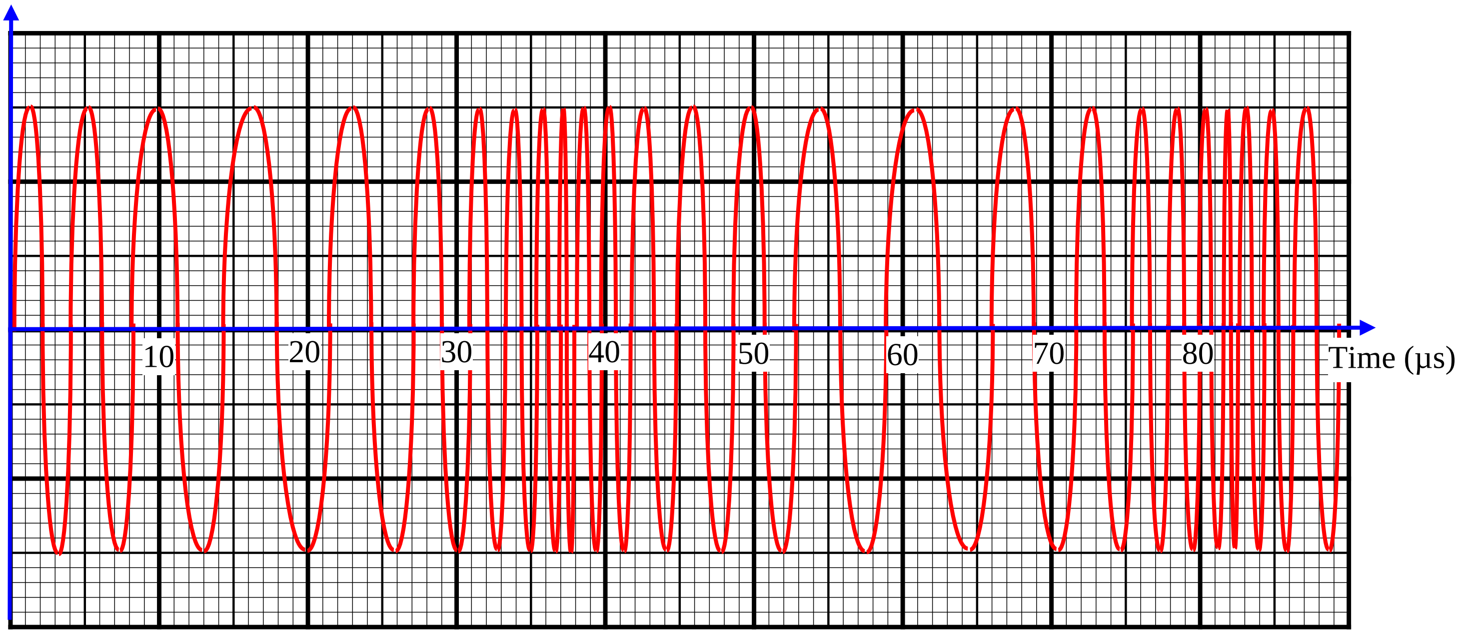
……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

[3]

4. The diagram below shows an FM carrier modulated by a pure tone (sinusoidal wave). Calculate the carrier frequency and the pure tone frequency.



....

....

....

....

[4]

Module ET4 – Communication Systems

Self Evaluation Review

Learning Objectives	My personal review of these objectives:		
	☺	😐	☹
sketch, recognise and analyse the resulting waveforms for a sinusoidal carrier being frequency modulated by a single frequency audio signal;			
recall that an FM-modulated carrier produces an infinite number of sidebands;			
recall that frequency deviation Δf_c is the maximum change in frequency of the carrier from its base value f_c;			
recall that the modulation index β is the FM equivalent to the depth of modulation;			
use the formula $\beta = \frac{\Delta f_c}{f_i}$, where f_i is the maximum frequency of the modulation signal;			
appreciate that almost all the power of a transmitted FM signal is contained within a bandwidth of $2(1+\beta) f_i$, where f_i is the maximum frequency of the modulating signal;			
recognise the frequency spectrum diagram for a sinusoidal carrier being frequency modulated by a single audio signal for β <1, β=1 and β =3.			
sketch, recognise and analyse the resulting waveforms for a sinusoidal carrier being frequency modulated by a single frequency audio signal;			

Targets: 1. ……………………………………………………………………………………………

……………………………………………………………………………………………

2. ……………………………………………………………………………………………

……………………………………………………………………………………………