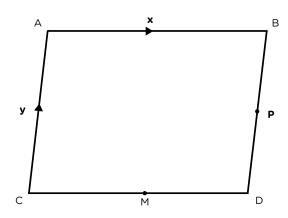# Vectors and geometric proofs Answers
GCSE Mathematics
HG25

1. In the parallelogram ABCD $\overrightarrow{AB}$ = **x** and $\overrightarrow{AC}$ = **y**.

M is the midpoint of CD and P is the midpoint of BD.
Find in terms of **x** and **y**:

a) $\overrightarrow{CD}$ *Answer:* $\overrightarrow{CD}$ = $\overrightarrow{AB}$

$\overrightarrow{CD}$ = x

b) $\overrightarrow{BD}$ *Answer:* $\overrightarrow{DB}$ = $\overrightarrow{CA}$

$\overrightarrow{DB}$ = y   so $\overrightarrow{BD}$ = -y

c) $\overrightarrow{CB}$ *Answer:* $\overrightarrow{CB}$ = $\overrightarrow{CA}$ + $\overrightarrow{AB}$

= y + x

d) $\overrightarrow{MP}$ *Answer:* $\overrightarrow{MP}$ = $\overrightarrow{MD}$ + $\overrightarrow{DP}$

= $\frac{1}{2}\overrightarrow{CD}$ + $\frac{1}{2}\overrightarrow{DB}$

= $\frac{x}{2}$ + $\frac{y}{2}$

e) What can you conclude about CB and MP?

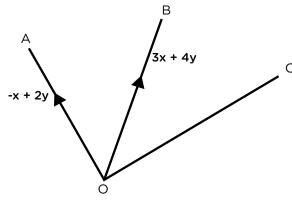*Answer:* As $\overrightarrow{MP}$ = $\frac{1}{2}\overrightarrow{CB}$ then the line MP is parallel to CB.

2. In the diagram $\overrightarrow{OA}$ = -**x** + 2**y** and $\overrightarrow{OB}$ = 3**x** + 4**y**.

B

A

3x + 4y

C

-x + 2y

O

(a) Find $\overrightarrow{AB}$ in terms of **x** and **y**

Answer: $\overrightarrow{AB}$ = $\overrightarrow{AO}$ + $\overrightarrow{OB}$

    = x - 2y + 3x + 4y

    = 4x + 2y

(b) Knowing that $\overrightarrow{BC}$ = 3**x** – **y** find $\overrightarrow{OC}$ in terms of **x** and **y**.

Answer: $\overrightarrow{OC}$ = $\overrightarrow{OB}$ + $\overrightarrow{BC}$

    = 3x + 4y + 3x - y

    = 6x + 3y

(c)   What does this tell us about the quadrilateral OABC?
Explain your answer.

Answer: $\overrightarrow{AB}$ = $\dfrac{2}{3}$ $\overrightarrow{OC}$ *therefore AB and OC must be parallel*

    OA and BC not parallel as no multiplier, therefore OABC is a trapezium.
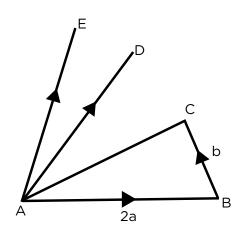
3.   In the following diagram

$\overrightarrow{AB}$ = 2**a** $\overrightarrow{BC}$ = **b**

(a)   Find $\overrightarrow{AC}$ in terms of **a** and **b**.

Answer: $\overrightarrow{AC}$ = $\overrightarrow{AB}$ + $\overrightarrow{BC}$

              = *2a* + *b*

(b)   Knowing that the point M is the midpoint of AD and
       $\overrightarrow{AM}$ = **a** + **b**, find $\overrightarrow{CD}$ in terms of **a** and **b**.

Answer: *If* $\overrightarrow{AM}$ =   *a + b then* $\overrightarrow{AD}$ = *2a + 2b*

       $\overrightarrow{CD}$ = $\overrightarrow{CA}$ + $\overrightarrow{AD}$

              =   *-2a - b + 2a + 2b*

              = *b*

(c)   Knowing that the point N is the midpoint of AE and
       $\overrightarrow{AN}$ = **a** + $^3/_2$**b**, find $\overrightarrow{DE}$ in terms of **a** and **b**.

Answer: *If* $\overrightarrow{AN}$ = *a +* $\frac{3\,b}{2}$ *then* $\overrightarrow{AE}$ *= 2* $\overrightarrow{AN}$
                                                 = 2a + 3b

       $\overrightarrow{DE}$ = $\overrightarrow{DA}$ + $\overrightarrow{AE}$

              = *-2a - 2b + 2a + 3b*

              = *b*

(d)   What conclusion can we draw about the points B,C,D and E?

Answer: $\overrightarrow{BC} = \overrightarrow{CD} = \overrightarrow{DE}$

$\overrightarrow{BD} = 2b$ and $\overrightarrow{BE} = 3b$

$\overrightarrow{BD} = 2\overrightarrow{BC}$ and $\overrightarrow{BE} = 3\overrightarrow{BC}$

Points BCDE must lie on a straight line