GCSE (9-1)

eduqas
Part of WJEC

WJEC Eduqas GCSE (9-1) in

MATHEMATICS

ACCREDITED BY OFQUAL

GUIDANCE FOR TEACHING

Teaching from 2015

wjec cbac

This Ofqual regulated qualification is not available for candidates in maintained schools and colleges in Wales.

INTRODUCTION

The WJEC Eduqas GCSE Mathematics qualification, accredited by Ofqual for first teaching from September 2015, is available to:

- all schools and colleges in England
- schools and colleges in independent regions such as Northern Ireland, Isle of Man and the Channel Islands
- independent schools in Wales.

It will be awarded for the first time in summer 2017, using grades 9 to 1.

Our GCSE Mathematics specification has two components.

The specification is tiered. Two tiers are available; foundation tier and higher tier.

The specification builds on the tradition and reputation WJEC has established for clear, reliable assessment supported by straightforward, accessible guidance and administration.

Key features include:

- Opportunities for flexible teaching approaches
- Straightforward wording of questions
- Accessibility of materials
- High-quality examination and resource materials

The full set of requirements is outlined in the specification which can be accessed on the Eduqas website.

In addition to this guide, support is provided in the following ways:

- Specimen assessment materials
- Face-to-face CPD events
- Examiners' reports on each question paper
- Free access to past question papers and mark schemes via the secure website
- Direct access to the subject officer
- Free online resources
- Exam Results Analysis
- Online Examination Review

AIMS OF THE TEACHER HANDBOOK

The principal aims of this Teacher Handbook are to offer support to teachers in delivery of the new WJEC Eduqas GCSE Mathematics specification and to offer guidance on the requirements of the qualification and the assessment process.

The guide is **not intended as a comprehensive reference**, but as support for professional teachers to develop stimulating and exciting courses tailored to the needs and skills of their own students in their particular institutions.

The guide contains detailed clarification and guidance on the subject content for both foundation and higher tiers.

The guide also contains a section on assessment objectives and how the different elements of these can be assessed in examination papers.

At the end of the guide you will find useful links to our free teaching and learning resources which includes printable and digital resources as well as our new question bank which allows users to generate their own revision papers using past paper questions.

1. SUBJECT CONTENT

All subject content within a particular tier (foundation and higher) can be assessed on either Component 1 (Non-calculator Mathematics) or Component 2 (Calculator-allowed Mathematics).

The subject content for both tiers is listed in the following pages.

The subject content has been grouped into the following topic areas:

- Number
- Algebra
- Ratio, proportion and rates of change
- Geometry and measures
- Probability
- Statistics

It is important that, during the course, learners should be given opportunities to:

- develop problem solving skills
- generate strategies to solve problems that are unfamiliar
- answer questions that span more than one topic area of the curriculum
- make mental calculations and calculations without the aid of a calculator
- make estimates
- understand 3-D shapes
- use computers and other technological aids
- collect data
- understand and use the statistical problem solving cycle.

This linear specification allows for a holistic approach to teaching and learning, giving teachers flexibility to teach topics in any order and to combine different topic areas.

This section gives the detail of the subject content. We also have included **further clarification** or **guidance** for the topics listed.

1.1 FOUNDATION TIER

- All learners at foundation tier will develop confidence and competence with the content identified by standard type.
- All learners at foundation tier will be assessed on the content identified by the standard and the underlined type; more highly attaining learners will develop confidence and competence with all of this content.

Note: Learners can be said to have confidence and competence with mathematical content when they can apply it flexibly to solve problems.

NUMBER

STRUCTURE AND CALCULATION

Code	Specification Statement	Further clarification or guidance
FN1.	order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥	Including converting from one form to another. The words ‘whole number’ and ‘integer’ may both be used. The symbol ≈ may be used to show an approximate relationship. [See also FN14, FN15]
FN2.	apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)	Questions may be in context. These essential skills underpin many other areas of the syllabus and are commonly combined with these areas. Learners should understand, for example: • the terms sum, difference, product and quotient • the relationship between a mixed number and an improper fraction • the structure of the decimal system and the relationship between places of decimal numbers [See also FN1, FN10, FR6, FR10]

FN3.	recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions) use conventional notation for priority of operations, including brackets, powers, roots and reciprocals	e.g. • $\frac{6}{7}\times\frac{35}{36}=\frac{{}^{1}\cancel{6}}{{}_{1}\cancel{7}}\times\frac{\cancel{35}^{5}}{\cancel{36}_{6}}=\frac{5}{6}$ • $\frac{2x+4}{2}=\frac{2x}{2}+\frac{4}{2}=x+2$ For example, understand, through using inverse operations, that: • $\left(\sqrt{3}\right)^2=3$ • $\sqrt[3]{(4)^3}=4$ and understanding that, for example, roots act to group calculations in the same way that brackets do: • $\frac{\sqrt{81+19}}{2}=\frac{\sqrt{100}}{2}=\frac{10}{2}=5$ [See also FN6, FA4]
FN4.	use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem	Write e.g. $60 = 2 \times 2 \times 3 \times 5$ or as $2^2 \times 3 \times 5$. Know that there is a unique way of writing a number as a product of its prime factors. [See also FN6]
FN5.	apply systematic listing strategies	For example, to find the HCF or LCM of a pair of integers or to list the possible outcomes of trials in an experiment. [See also FN4, FP1, FP7]

FN6.	use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5	Examples: • simplify $5 \times 5 \times 5 \times 5$ to 5^4 • write down the value of 4^2 as 16 • write down the value of $\sqrt[3]{125}$ as 5 • write down the value of 30^2 as 900 [See also FN4]
FN7.	<u>calculate with roots</u>	Examples: • work out the value of $3+\sqrt[3]{64}$ as 3 + 4 = 7 • find the value of $\frac{\sqrt{256}}{4}$ as $\frac{16}{4}=4$ • simplify $4\sqrt{3}-\sqrt{3}$ as $3\sqrt{3}$ [See also FG18, FA4]
	<u>and with integer indices</u>	Know and use the rules of indices for positive and negative integers. Examples: • find the value of $5^{-4}\times5^{6}$ as $5^{-4+6}=5^{2}=25$ • work out the value of 10^{-2} [See also FN9, FA1]

FN8.	calculate exactly with fractions and multiples of π	Learners should know that writing a fraction in its simplest form or in its lowest terms means writing the numerator and denominator as whole numbers that are as small as possible. For most fractions, this will mean cancelling down, but occasionally, if a numerator or denominator is a decimal, learners may need to multiply. [See also FN2] Questions requiring an answer as a multiple of π (i.e. in terms of π) will commonly be assessing the mensuration of a circle or cylinder or composite shape formed from one of these. For example, finding the area of a circle with radius 6 cm as 36π cm^2. Note that, the question will indicate when answers are required in this form. [See also FG14. FG15, FG16]
FN9.	calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer	Convert numbers between ordinary form and standard form and vice versa. Calculation with numbers in standard form may be with or without the use of a calculator. Calculations may involve one or more of the four operations. Questions may be in context. [See also FN7]

FRACTIONS, DECIMALS AND PERCENTAGES

Code	Specification Statement	Further clarification or guidance
FN10.	work interchangeably with terminating decimals and their corresponding fractions (such as 3·5 and $\frac{7}{2}$ or 0·375 and $\frac{3}{8}$)	For example: • Use place value to convert terminating decimals to fractions. • Convert a fraction with a denominator that is a factor of 100 into a decimal using equivalent fractions. • Convert a simple fraction to a decimal using a division algorithm. [See also FN1, FN2]
FN11.	identify and work with fractions in ratio problems	For example, understand that: • a ratio of juice to water = 1 : 3 in a drink means that juice is ¼ of the drink. • if the ratio of boys to girls in a class is 3 : 4 then $\frac{3}{7}$ of the class is boys and $\frac{4}{7}$ is girls. [See also FN12]
FN12.	interpret fractions and percentages as operators	Calculating a fraction of a quantity or a percentage of a quantity. May be with or without a calculator. Questions may be in context. May involve converting to a decimal multiplier. Underpinning skill, for example, that multiplying by ½ has the same result as dividing by 2 or into 2 equal parts. [See also FR10]

Code	Specification Statement	Further clarification or guidance
FN13.	use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate	Including converting between common metric units and use these conversions when solving problems. [See also FN16, FR1, FR2]
FN14.	estimate answers; check calculations using approximation and estimation, including answers obtained using technology	A useful working rule is to encourage learners to round values in any estimation of a calculation to 1 significant figure. Calculations should be able to be done easily by mental calculation if the estimate is successful. Calculations may involve powers and roots. e.g. James calculates $(8.099 \times 0.542)^2$ as 2.64986.... Determine, by estimation, whether or not James is correct. You must show all your working. Leading to the result $(8 \times 0.5)^2 \approx 4^2 = 16$, so “not correct as the answer must be more than 16”. Note that: • the determination (i.e. “correct” or “not correct” in this example) must be explicitly stated • justification may not always be required

FN15.	round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding	Appropriate degrees of accuracy are, for example: • 2 decimal places for calculations involving money • no greater accuracy than the values given in the question – so if the given values are stated to 2 decimal places, the final answer should be given to no more than 2 decimal places Learners should ensure that values in their working are given to a greater accuracy than that to which the final answer is required i.e. they should not prematurely approximate working values. e.g. • The mass of an object is 200 grams, correct to the nearest 10 grams. Complete the statement about the mass, *m* grams, of the object. .... $\leqslant m <$ .... Answers with 195 and 205. • A height of 164 cm has been truncated to the nearest integer. Complete the statement about this height. .... $\leqslant$ height $<$ .... Answers with 164 and 165. Note that learners may be required to form an inequality statement rather than simply completing it. [See also FN1, FN16]

FN16.	<u>apply and interpret limits of accuracy</u>	Learners should know that measurement is inaccurate and be able to state, in simple cases, what the limits of that accuracy are. Interpretation will commonly be in the form of simple problem solving. For example, • understanding that when the mass of an item has been stated as 5 kg to the nearest kilogram, the mass could be more or less than 5 kilograms and apply that information to solve a problem • explaining with justification that, if Jenny is 165 cm tall, to the nearest cm, she may be too short to ride on a fairground ride where the minimum ride height is 165 cm as she might actually only be 164.5 cm tall. Formal knowledge of bounds is not required. [See also FN15]

ALGEBRA

NOTATION, VOCABULARY AND MANIPULATION

Code	Specification Statement	Further clarification or guidance
FA1.	use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets	[See also FN7, FN12]
FA2.	substitute numerical values into formulae and expressions, including scientific formulae	Formulae and expressions will be given in such questions. Numbers may be positive or negative and may not be integers. [See also FA5, FA19]
FA3.	understand and use the concepts and vocabulary of expressions, equations, formulae, <u>identities</u>, inequalities, terms and factors	Learners should understand that, in an identity, the left hand side is always equal to the right hand side, regardless of the values of the variables. They should also know that identities are not 'solved' as equations are. For example, given the identity $3x + by \equiv ax - 2y$, state the values of the constants a and b as 3 and -2. [See also FA2, FA4, FA6]

FA4.	simplify and manipulate algebraic expressions (including those involving surds) by: • collecting like terms • multiplying a single term over a bracket • taking out common factors • expanding products of two binomials • factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares • simplifying expressions involving sums, products and powers, including the laws of indices	Including, for example, questions involving surds such as: • Simplifying $2a+\sqrt{5}-a-3\sqrt{5}$ as $a-2\sqrt{5}$ • Expanding $4\left(x-\sqrt{2}\right)$ as $4x-4\sqrt{2}$ • Expanding $\left(1-\sqrt{2}\right)\left(3+\sqrt{2}\right)$ as $3+\sqrt{2}-3\sqrt{2}-2$ $=1-2\sqrt{2}$, (using FOIL method or similar). Note that instructions such as "Factorise" or "Simplify" should always be taken as factorising or simplifying as far as possible. [See also FN3, FN7, FA17, FG19]

FA5.	understand and use standard mathematical formulae;	Formulae may be in words or symbols. Formulae which must be recalled [See specification: Appendix A]: • Circumference of a circle = $2\pi r$ or πd • Area of a circle = πr^2 • Pythagoras' theorem • Trigonometric formula for sine, cosine and tangent of an angle • Volume of a prism, area of a trapezium and other general perimeter, area and volume formulae • Compound interest: Total accrued = $P\left(1+\frac{r}{100}\right)^n$
	rearrange formulae to change the subject	Formulae to be rearranged to change the subject include those where the new subject appears twice in the formula, or where the formula involves a simple power, root or reciprocal. e.g. • Rearranging $A=\frac{100}{h}$ to make h the subject as $h=\frac{100}{A}$ • Rearranging $A=\pi r^2$ to make r the subject as $r=\sqrt{\frac{A}{\pi}}$ • Rearranging $ax-b=x+2$ to make x the subject as $ax-x=b+2 \rightarrow x(a-1)=b+2 \rightarrow x=\frac{b+2}{a-1}$ [See also FN3]

FA6.	<u>know the difference between an equation and an identity;</u> <u>argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments</u>	As previously stated, learners should be aware that identities cannot be 'solved' and that terms should not be moved from the left to the right in, for example, a simple proof. e.g. • Prove $(x+1)(x-1)+3 \equiv x^2+2$ by expanding the left hand side and collecting terms: $x^2 \cancel{-x} \cancel{+x} -1+3 = x^2+2$ as required. [See also FA2, FA3, FA4]
FA7.	where appropriate, interpret simple expressions as functions with inputs and outputs	Including using simple number or function machines. [See also FN3]

GRAPHS

Code	Specification Statement	Further clarification or guidance
FA8.	work with coordinates in all four quadrants	In 2 dimensions only.
FA9.	plot graphs of equations that correspond to straight-line graphs in the coordinate plane;	Note that: • learners may observe that, even though $y = mx + c$ is commonly referred to as the general *equation* of a straight line, it is in fact used as a *formula* with input x and output y (rather an equation to be solved) • this includes straight lines parallel to the coordinate axes
	use the form $y = mx + c$ to identify parallel lines;	May be combined with changing the subject of a formula, so learners may need to rearrange, for example, $3y + 6x = 10$.
	find the equation of the line through two given points,	Information may be presented graphically or points may be given in coordinate form. Learners should draw their own simple diagram in the latter case.
	or through one point with a given gradient	Again, learners should make a sketch to help them if a diagram has not been given. [See also FA10]
FA10.	identify and interpret gradients and intercepts of linear functions graphically and algebraically	Gradients may be positive, negative or rational. Includes understanding that the gradient represents the rate of change of y as x changes. [See also FA9, FA13, FR15]

FA11.	<u>identify and interpret roots, intercepts, turning points (stationary points) of quadratic functions graphically;</u> <u>deduce roots algebraically</u>	For example, learners: • may be given a graph or may need to draw the graph first • should be aware of the symmetrical nature of the curve • should be able to identify the equation of the line of symmetry of the curve [See also FA9, FA10, FA12] [See also FA15]
FA12.	recognise, sketch and interpret graphs of linear functions, quadratic functions, <u>simple cubic functions, the reciprocal function</u> $y=\frac{1}{x}$, <u>with</u> $\underline{x \neq 0}$	For example, • identify the graph of $y=\frac{1}{x}$ from a selection of 6 graphs, • sketch the graph of $y=x^3-1$ for $-3 \leq x \leq 3$, • sketch the graphs of $y=3x$ and $y=2x^3$, for $-2 \leq x \leq 2$ and write down the number of times the graphs intersect. [See also FA9, FA10, FA11, FA13, FA16]
FA13.	plot and interpret graphs (<u>including reciprocal graphs</u>) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration	Includes, for example, travel graphs and conversion graphs. [See also FA10, FA12, FR11]

SOLVING EQUATIONS AND INEQUALITIES

Code	Specification Statement	Further clarification or guidance
FA14.	solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation);	This skill is commonly combined with manipulation techniques so, for example, equations may include multiplying a single term over a bracket. Trial and improvement is a numerical not an algebraic method and should not be used. Learners may need to form equations from problems. [See also FA17]
	find approximate solutions using a graph	Occasionally learners may need to demonstrate interpretation of the graphs of linear functions. Graphs may first need to be drawn or may be given. For example, • solving $5x-2=0$ graphically by writing down the root • solving $4-3x=5$ graphically by reading off the value of x when $y = 5$.
FA15.	solve quadratic equations of the form $x^2 + bx + c$ (NOT including those that require rearrangement) algebraically by factorising;	Underpinning skills, for example, are: • writing down the factors of an integer • multiplication of directed numbers • expanding binomial brackets Trial and improvement is a numerical not an algebraic method and should not be used. c could be equal to 0. Learners may need to form equations from problems. [See also FN4, FA4, FA17]
	find approximate solutions using a graph	For example, understanding that the solutions (roots) of the equation $x^2 + bx + c = 0$ are the x-intercepts of the graph. Graphs may first need to be drawn or may be given. [See also FA11, FA12]

FA16.	solve two simultaneous linear equations in two variables algebraically; find approximate solutions using a graph	Trial and improvement is a numerical not an algebraic method and should not be used. Learners may use the process of substitution or elimination and should be encouraged to check that their solution satisfies both original equations. Equations may need to be derived from problems and may involve working with decimal quantities in context. [See also FA17] Graphical methods of solution should only be used when instructed. Learners should understand that the coordinates of the unique point of intersection of the two lines give the value of each of the unknowns which satisfy the equations represented. [See also FA9, FA12]
FA17.	translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution	For example: • A square with side x cm has an area of $36\,\text{cm}^2$. Form an equation and solve it to find the length of the side of the square. Leading to the result $x^2 = 36,\ x = 6$ only. • The sum of three consecutive numbers is 84. Using n as the smallest of the three numbers, form and solve an equation in n and state the three numbers that were summed. Leading to the result $n + n + 1 + n + 2 = 84$, $3n + 3 = 84$, $n = 27$. The three numbers were 27, 28, 29. At this level simultaneous equations will both be linear. [See also FA16]

FA18.	solve linear inequalities in one variable;	Learners should: • know that the solution may be a finite or an infinite set of values and not a single value • present their solution using correct notation For example, • solving $3x-4<8$ as $x<4$ • solving $6<3x\le 21$ as $2<x\le 7$ • solving $-2<3x+1\le 5$ as $-1<x\le \frac{4}{3}$ [See also FN1]
	represent the solution set on a number line	Learners should know and use the convention of an empty circle to represent < or > and a full circle to represent ≤ or ≥. The number line may need to be drawn.

SEQUENCES

Code	Specification Statement	Further clarification or guidance
FA19.	generate terms of a sequence from either a term-to-term or a position-to-term rule	At this level term-to-term rules will not be expressed in formal subscript notation. Learners should be able to: • continue a sequence given a rule or by spotting a pattern • use a given expression for the *n*th term of a sequence to find a given term. Given expressions for *n*th terms may not be linear. [See also FN2, FA2, FA20]
FA20.	recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, <u>Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0)</u>	For example, writing down the next two terms of each of these sequences: • –1, –3, –5, –7, ... as –9, –11 • 5, –3, 2, –1, 1, ... as 0, 1 • 96, 48, 24, 12, 6, ... as 3, 1.5 • 2, 5, 10, 17, 26, ... as 37, 50 [See also FN2, FA19, FA21]
FA21.	deduce expressions to calculate the *n*th term of linear sequences	Including finding sequences from patterns in diagrams For example, find the *n*th term of: • 4, 8, 16, 24, ... as $4n$ • –1, –3, –5, –7, ... as $1 - 2n$ • ½, 1½, 2½, 3½, ... as $n - ½$ [See also FN2, FA20]

Code	Specification Statement	Further clarification or guidance
FR1.	change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts	Metric unit conversions and time unit conversions should be known. Imperial unit conversions will be stated in the question if needed. This may be embedded in another question and assessed indirectly. This may involve converting compound units in algebraic expressions. For example, John travels at $6x$ km/h. What is John's speed in metres per minute? Give your answer as an expression in x in its simplest form.
FR2.	understand the concept of density and be able to use the relationship between density, mass and volume; understand the concept of pressure and be able to use the relationship between pressure, force and area	Know and use: • The units of density, for example, a mass in kg and volume in m^3 results in a density of kg per m^3 • Understand that the density of material is the mass of an object for 1 unit of its volume and interpret kg per m^3 and g per cm^3 in this way. • $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ Know and use : • The units of pressure, for example, a weight in Newtons and an area in m^2 results in a pressure of N per m^2 • Understand that pressure is the force produced when something presses or pushes against something else and is the force applied to each unit of area and interpret N per m^2 in this way. • $\text{Pressure} = \frac{\text{Force}}{\text{Area}}$ It is important that learners understand the difference between mass and weight (weight being a force).and appreciate that a mass of 1 kg has a weight of approximately 10 Newtons.

		Questions on this topic will commonly involve comparisons of densities or pressures. [See also FR1, FR12]
FR3.	use scale factors, scale diagrams and maps	May commonly be combined with bearings. [See also FG13]
FR4.	express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1	
FR5.	use ratio notation, including reduction to simplest form	Learners should know that reducing to simplest form means writing all the values in the ratio with whole numbers that are as small as possible. A ratio should not have decimal parts nor should it have units stated.
FR6.	divide a given quantity into two parts in a given part:part or part:whole ratio; divide a given quantity into more than two parts; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)	
FR7.	express a multiplicative relationship between two quantities as a ratio or a fraction	For example, • if the amount of flour in a recipe is double the amount of butter, the ratio of flour to butter is 2 : 1 and there is ½ as much butter as there is flour in the mixture. • if the amount of sugar in a recipe is 1½ times the amount of fruit then the ratio of sugar to fruit is 1½ : 1 or 3 : 2

FR8.	understand and use proportion as equality of ratios	Learners should understand, if *a* is to *b as c* is to *d,* then the proportion equation $\frac{a}{b}=\frac{c}{d}$ can be formed and also $a : b = c : d$. For example, • 1 is to 2 as 3 is to 6 and so $\frac{1}{2}=\frac{3}{6}$ and also 1 : 2 = 3 : 6 • if $3 : x = 8 : 24$ then $\frac{3}{x}=\frac{8}{24}$ and so $x = 9$ (using equivalent ratios, equivalent fractions or solving for *x*) This approach is commonly used to find missing lengths in similar figures. [See also FR13]
FR9.	relate ratios to fractions and to linear functions	Includes, for example, • increasing a value in the ratio 3 : 2 by multiplying it by $\frac{3}{2}$ • decreasing a value in the ratio 1 : 4 by multiplying it by $\frac{1}{4}$ • understanding that, when for example, $x:y=1:2$, then $\frac{x}{y}=\frac{1}{2}$ which means that the points (x, y) which are in the ratio $x:y=1:2$ lie on the line $y=2x$. [See also FN11, FR7, FR8, FR15]

FR10.	define percentage as 'number of parts per hundred';	
	interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively;	Learners should be able to convert a percentage or a percentage change into a simple fractional or decimal multiplier and use this in problem solving. For example, understand that: • increasing an amount by 50% means multiplying by 1.5 or $\frac{150}{100}$ • decreasing a quantity by 25% means multiplying by 0.75 or $\frac{3}{4}$
	express one quantity as a percentage of another;	
	compare two quantities using percentages;	Includes Best Buy comparisons.
	work with percentages greater than 100%;	
	solve problems involving percentage change, including percentage increase / decrease and original value problems, and simple interest including in financial mathematics	Includes profit and loss and discounts e.g. • A phone costs £102 in a 15% off sale. How much did the phone cost before the sale? Leading to a solution of $\frac{85}{100}$ of original cost = £102 $\text{original cost} = £\frac{102}{85} \times 100 = £120$

FR11.	solve problems involving direct and inverse proportion, including graphical and algebraic representations	Traditional word problems, such as recipes, best buys and exchange rates. [See also FR10] Learners should know that quantities x and y are in: • direct proportion if $\frac{y}{x}$ is a constant value • inverse proportion if xy is a constant value e.g. • determining that for ordered pairs (2, –4), (3, –6), (5, –10), (10, –20), x and y are in direct proportion as $\frac{y}{x}$ is always –2 and the graph plotted would be that of $y=-2x$ • knowing that the graph of $y=\frac{20}{x}$ represents quantities in inverse proportion as $xy=20$ Note that a constant of proportion may be negative (as shown above), so care should be taken not to confuse learners by suggesting that quantities are in inverse proportion if, when one increases the other decreases, for example. [See also FA5]
FR12.	use compound units such as speed, rates of pay, unit pricing, <u>density and pressure</u>	Learners should understand compound units and know that, for example, • a speed of 60 mph means travelling 60 miles in one hour • a pay rate of £6.90 per hour means earning £6.90 for every hour worked • a price of £3 per 100ml means that every 100 ml of a product costs £3 [See also FR1, FR2, FR11]
FR13.	compare lengths, areas and volumes using ratio notation; <u>make links to similarity (including trigonometric ratios)</u> and scale factors	[See also FR8]

FR14.	understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; interpret equations that describe direct and inverse proportion	Learners should know and be able to use: • If y is (directly) proportional to x the relationship $\frac{y}{x} = k$ holds, where *k* represents the constant, and so $y = kx$ • If y is inversely proportional to x the relationship $xy = k$ holds, where *k* represents the constant, and so $y = \frac{k}{x}$ Note that when quantities are in direct proportion, the word "direct" may or may not be stated. When quantities are in inverse proportion, this will be indicated. [See also FA5, FA12, FR9, FR11]
FR15.	interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion	Includes interpreting the gradient as a rate of change for linear functions – for example • in a distance-time graph the gradient of a straight line segment would represent the rate of change of the distance as the time changes e.g. time (seconds) on the horizontal axis and distance (metres) on the vertical axis would result in a gradient which represented the number of metres per second i.e. the speed • in a currency conversion graph the gradient would represent the exchange rate e.g. pounds on the horizontal axis and dollars on the vertical axis would result in a gradient which represented the number of dollars per pound [See also FA10, FA13, FR9] [See also FA12]
FR16.	set up, solve and interpret the answers in growth and decay problems, including compound interest	Including appreciation and depreciation problems. [See also FA5]

GEOMETRY AND MEASURES

PROPERTIES AND CONSTRUCTIONS

Code	Specification Statement	Further clarification or guidance
FG1.	use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description	 This may be assessed directly or indirectly as part of a geometry question. Learners will need to recall properties of special triangles, quadrilaterals and regular shapes, including symmetry. [See also FG4]
FG2.	<u>use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle);</u> <u>use these to construct given figures and solve loci problems;</u> <u>know that the perpendicular distance from a point to a line is the shortest distance to the line</u>	Learners will be expected to be familiar with the language, for example: • perpendicular • bisector/bisects • mid-point • equidistant • locus/path • shade the region that satisfies • When accurate drawings are asked for, compass constructions are expected. For example, mid-points should be constructed not measured. May be combined with trigonometry and/or Pythagoras' theorem.

FG3.	apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)	When reasons are required, it is expected that correct mathematical language should be used, rather than, for example, F-angles or Z-angles. Interior angle sum of any triangle = 180° to make deductions including • Interior angle sum of any n-sided polygon = $180(n-2)°$ • Exterior angle sum of any polygon = 360° • Interior angle size of any regular n-sided polygon as $\frac{180(n-2)}{n}$ or as $180-\frac{360}{n}$ [See also FG4]
FG4.	derive and apply the properties and definitions of: special types of triangles, quadrilaterals (including square, rectangle, parallelogram, trapezium, kite and rhombus) and other plane figures using appropriate language	Learners should know the properties of these triangles and know their mathematical names: equilateral, isosceles, right-angled, scalene Other plane figures include, for example: pentagon, hexagon, octagon, decagon Learners may need to demonstrate the validity of a given statement by presenting an argument justified by reasons. [See also FG3]
FG5.	<u>use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)</u>	Triangles may have common sides. [See also FG6, FG7]
FG6.	<u>apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs</u>	Learners may need to demonstrate the validity of a given statement by presenting an argument justified by reasons. [See also FG3, FG4, FG5]

FG7.	identify, describe and construct congruent and similar shapes,	Learners should know that: • the criterion AA is enough to show two triangles are similar (as AAA follows) • for other similar shapes, sides must be in proportion as well as angles being invariant [See also FG5]
	including on coordinate axes, by considering rotation, reflection, translation and enlargement (<u>including fractional scale factors</u>)	Learners should know that: • enlargement produces similar shapes • rotation, reflection and translation are examples of rigid motion and produce congruent shapes [See also FG20]
FG8.	identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, <u>tangent, arc, sector and segment</u>	[See also FN8, FG16]
FG9.	solve geometrical problems on coordinate axes	
FG10.	identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres	
FG11.	<u>construct and</u> interpret plans and elevations of 3D shapes	Including using isometric paper.

Code	Specification Statement	Further clarification or guidance
FG12.	use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)	[See also FR1]
FG13.	measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings	Using a ruler and protractor. When a reflex angle is required, this will be indicated in the question, otherwise learners should assume the angle to be measured is between 0° and 180°. Bearings will be given and expected as 3 figures. Compass points (4 major N, E, S, W and 4 minor NE, SE, SW, NW) will also be used and expected. Bearings may be measured or calculated, but should only be measured if the diagram given is drawn to scale. [See also FR3, FG3, FG18]
FG14.	know and apply formulae to calculate: area of squares, rectangles, triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)	Whilst cylinders are technically not right prisms, they are included here as they have the same properties for the calculation of volume. [See also FA5]
FG15.	know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids	The formulae for cones and spheres (surface area and volumes) will be given and need not be recalled. Composite solids may be, for example, the frustum of a cone or pyramid, a hemi-sphere and so on. [See also FN8, FA5]
FG16.	calculate arc lengths, angles and areas of sectors of circles	At this level, applications relating to arcs, sectors and segments are likely to be simple cases where the angle subtended at the centre is a factor of 360°, resulting in the sector under consideration being a unit fraction of the circle. [See also FN8, FG8]

FG17	apply the concepts of congruence and similarity, including the relationships between lengths in similar figures	[See also FR8, FR11, FG5, FG6, FG7]
FG18.	know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$, $\tan\theta = \frac{opposite}{adjacent}$; apply them to find angles and lengths in right-angled triangles in two dimensional figures	For trigonometric questions, learners should be careful to make sure that their calculators, if appropriate, are in degrees. It is possible that questions will be set where learners are not permitted to use a calculator. Questions which involve simple multi-step solutions may be set. Pythagoras' theorem may be used to find the distance between two points on a coordinate grid. At this level a supporting diagram will be given. [See also FN7, FG19]
FG19.	know the exact values of $\sin\theta$ and $\cos\theta$ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of $\tan\theta$ for θ = 0°, 30°, 45° and 60°	Learners may find it helpful to consider an equilateral triangle of side 2 and an isosceles right angled triangle with equal sides 1. [See also FA4, FG4, FG18]

VECTORS

Code	Specification Statement	Further clarification or guidance
FG20.	describe translations as 2D vectors	For example, describing a triangle that has been mapped from coordinates (0, 0), (1, 3), (4, 5) to (-1, 2), (0, 5), (3, 7) as a translation by the vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ [See also FG7, FG21]
FG21.	<u>apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors</u>	Vectors will be 2D. Includes drawing vectors on a (coordinate) grid. Notation used will be, for example: • **OA** • **a** • 2**b** For example, given **a** = $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and **b** = $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ finding 2**a** + **b** as $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$, finding –**b** as $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

PROBABILITY

Code	Specification Statement	Further clarification or guidance
FP1.	record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees	Probabilities should be given as decimals, fractions or percentages, but **not** as ratios.
FP2.	apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments	Expected number or expected frequency. Underpinning skill is multiplication of an integer by a decimal or fraction. [See also FN2]
FP3.	relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale	
FP4.	apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one	Use P(Not A) = 1 – P(A) Understand the meaning of mutually exclusive.
FP5.	<u>understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size</u>	Understand that relative frequency tends towards theoretical probability as sample size increases. For example, observing that, the more times a fair coin is flipped the closer the relative frequency of the number of heads gets to the theoretical probability of ½ . [See also FP1]
FP6.	enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams <u>and tree diagrams</u>	i.e. to systematically list all the elements in a set. At this level, Venn diagrams will be 2 circle only. Ɛ will be used for the Universal set. Formal set notation is not required at Foundation Tier. Learners may need to draw their own table, grid, tree diagrams or Venn diagrams. [See also FN5]

FP7.	construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities	Possibility spaces may be, for example, numbers in a grid or points marked in a Cartesian plane and are a list of all possible outcomes. For example, • constructing a table of possible outcomes when two spinners numbered 1, 2, 3, 4 are spun together and the results summed. • using the possibility space (table) constructed to work out the probability that the total is less than 4, by counting the outcomes less than 4 and writing them as a fraction out of the total number of possible outcomes. [See also FP6]
FP8.	<u>calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions</u>	Learners should: • understand informally when to add probabilities and when to multiply probabilities • understand, through example, that when events are not mutually exclusive, some outcomes may be counted twice and so an adjustment needs to be made to correct this. An example would be selecting a Jack or a red card from an ordinary pack of 52 playing cards. Two Jacks are red and so the number of cards is 26 + 4 – 2 = 28 and the probability is therefore $\frac{28}{52}$. [See also FN2] • be encouraged to draw tree diagrams for dependent combined events to ensure that probabilities used are correct. Other representations may include Venn diagrams and simple tables. Underpinning skills are addition and multiplication of decimals and/or fractions [See also FN2, FP6]

Code	Specification Statement	Further clarification or guidance
FS1.	infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling	Learners should: • understand the difference between a simple population and a sample • understand the problems associated with surveying a whole population and why taking a sample and estimating results is preferable • understand the concept of taking a simple random sample and the concept of bias • know that the greater the sample size the more reliable the results i.e. small samples do not produce reliable results. Also includes using the proportion of successes in a sample to infer the size of a population, for example, using the capture/ recapture method. For example, to estimate the number of a type of bird in a forest, a scientist captures 50 birds, rings their leg and lets them go. The following day, the scientist captures 30 birds and 5 of them are ringed. This should lead to the inference that as one sixth of the birds are ringed, the original 50 birds represents one sixth of the population of birds. Simple assumptions made when making such inferences and the impact they have on the result obtained should also be considered. For example, the population may decrease through deaths or increase through births between samples being taken; the sample may not be taken in a random way. [See also FS2, FR8]
FS2.	designing and criticising questions for a questionnaire, including notion of fairness	[See also FS1]

FS3.	interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use	Includes selection of an appropriate representation for data set/sets. Bar charts may be simple, dual or stacked. Use time series data to identify trends such as seasonal variations.
FS4.	interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data • appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outlier)	Includes understanding the difference between discrete and continuous data and the reasons for grouping data
FS5.	apply statistics to describe a population	

FS6.	use and interpret scatter graphs of bivariate data; recognise correlation <u>and know that it does not indicate causation;</u>	Learners should understand that correlation does not imply causation i.e. that the change in one data set has caused the change in the other data set. There may be other influences acting on each data set. For example, the number of bottles of sunscreen sold at a seaside resort and the number of cold drinks sold at the resort each day in July may have a positive correlation. However, learners should be able to reason that it is highly unlikely that the increase in sales of bottles of sunscreen is the cause of the increase in sales of cold drinks. It is more likely that July being an increasingly hot month is the reason for each of these increases.
	<u>draw estimated lines of best fit;</u>	If the mean of each data set is given/ calculated, learners should know that the line of best fit passes through that point. Learners should know a line of best fit must be a single straight line drawn with a ruler. The instruction to draw "by eye" does not mean without a ruler. It means that the line does not need to go through the mean point.
	<u>make predictions;</u>	Using a line of best fit.
	<u>interpolate and extrapolate apparent trends whilst knowing the dangers of so doing</u>	Learners should know that lines of best fit are not always appropriate to estimate values and that: • interpolating is relatively safe as the estimation is within the data set given and so it is reasonable to use the line of best fit given or drawn • extrapolation is not safe as no information should be assumed about data that is not within the information that is known – and the further the extrapolated point is away from the known data, the more inappropriate it may be to use it as a predictor of values.

1.2 HIGHER TIER

- All learners at higher tier will develop confidence and competence with the content identified by standard type.
- All learners at higher tier will be assessed on the content identified by the standard and the <u>underlined</u> type; more highly attaining learners will develop confidence and competence with all of this content.
- The highest attaining learners will develop confidence and competence with the **bold** content.

Note: Learners can be said to have confidence and competence with mathematical content when they can apply it flexibly to solve problems.

Please note that the clarification and guidance in this section mostly applies to the **bold** *content only. Please refer to section 1.1 for clarification and guidance on the content in* standard *type or* <u>underlined</u> *type.*

NUMBER

STRUCTURE AND CALCULATION

Code	Specification Statement	Further clarification or guidance
HN1.	order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥	The symbol ≈ may be used to show an approximate relationship.
HN2.	apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)	
HN3.	recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions) use conventional notation for priority of operations, including brackets, powers, roots and reciprocals	
HN4.	use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem	

HN5.	apply systematic listing strategies **including use of the product rule for counting**	Learners should be able to apply the product rule for counting to various situations. For example, these may involve linear arrangements of objects, sometimes with simple diagrams, consideration of the formation different types of number or consideration of practical problems. For example, if you had 3 options for a starter, 4 for a main and 5 for dessert, then there would be 3 × 4 × 5 = 60 different combinations of options altogether. Learners are expected to apply the product rule for counting rather than trying to list all possible outcomes and the number of outcomes possible will usually preclude learners from listing.
HN6.	use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5; **estimate powers and roots of any given positive number**	For example, without a calculator, it is expected that learners will be able to use knowledge of powers and roots to estimate numbers, e.g. $6{\cdot}2^3$, $\sqrt{630}$, $\sqrt[4]{100}$
HN7.	calculate with roots, and with integer **and fractional** indices	For example, without a calculator, find the value of • $8^{\frac{2}{3}}$, • $9^{-\frac{5}{2}}$ • $\left(81^{\frac{1}{4}}\right)^{-1}+\left(\sqrt[6]{27}\right)^2$ [See also HN8]

HN8.	calculate exactly with fractions, **surds** and multiples of π**;** **simplify surd expressions involving squares** **(e.g. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators**	Simplification may involve the use of • algebra e.g. expand and simplify $(3+\sqrt{2}x)(3-\sqrt{5}x)$ or • geometry, such as finding the area of a triangle with an irrational base and height. Rationalising includes monomial and binomial denominators, e.g. $\frac{14}{\sqrt{7}}, \frac{3}{4+\sqrt{5}}$ and may also be embedded in problem with an algebraic or geometric context. [See also HA4]
HN9.	calculate with and interpret standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer	At this level, questions are more likely to be set in context or the skills required may be embedded in another question and assessed indirectly.

FRACTIONS, DECIMALS AND PERCENTAGES

Code	Specification Statement	Further clarification or guidance
HN10.	work interchangeably with terminating decimals and their corresponding fractions (such as 3·5 and $\frac{7}{2}$ or 0·375 and $\frac{3}{8}$); **change recurring decimals into their corresponding fractions and vice versa**	Learners will be expected to use correct notation for recurring decimals. For example, • express $5\cdot4\dot{3}\dot{2}$ as a fraction, • show that $0\cdot\dot{5}1\dot{2}$ can be written as $\frac{512}{999}$ express $\frac{2}{11}$ as a decimal.
HN11.	identify and work with fractions in ratio problems	
HN12.	interpret fractions and percentages as operators	

MEASURES AND ACCURACY

Code	Specification Statement	Further clarification or guidance
HN13.	use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate	
HN14.	estimate answers; check calculations using approximation and estimation, including answers obtained using technology	
HN15.	round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding	
HN16.	apply and interpret limits of accuracy**, including upper and lower bounds**	Greatest and least values, or upper and lower bounds, should not be confused with rounding. For example, 40 litres correct to the nearest 5 litres has a lower bound of 37.5 litres and an upper bound of 42.5 litres. The upper bound is not, for example, 42.49 and 42.5 is the only acceptable upper bound. It is expected that learners will solve problems involving bounds including any of the 4 operations (add, subtract, multiply or divide). For example, find the greatest possible average speed, in km/h, where distance is given correct to the nearest 10 km and time is given correct to the nearest 10 minutes. [See also HR1]

ALGEBRA

NOTATION, VOCABULARY AND MANIPULATION

Code	Specification Statement	Further clarification or guidance
HA1.	use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets	
HA2.	substitute numerical values into formulae and expressions, including scientific formulae	
HA3.	understand and use the concepts and vocabulary of expressions, equations, formulae, <u>identities</u>, inequalities, terms and factors	

HA4.	simplify and manipulate algebraic expressions (<u>including those involving surds</u> **and algebraic fractions**) by: • collecting like terms • multiplying a single term over a bracket • taking out common factors	Simplification of algebraic fractions may mean combining two or more fractions as a single fraction or factorising the numerator and denominator of a single fraction and cancelling common factors. Learners may also be asked to solve equations involving terms such as $\frac{3x+4}{2x-3}-\frac{5x-6}{2x+5}$.
	• <u>expanding products of two</u> **or more** <u>binomials</u>	This may involve finding expressions such as $(1+x)^3$ by multiplying out $(1+x)(1+x)(1+x)$. The use of the binomial theorem is not required.
	• <u>factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares</u>; **factorising quadratic expressions of the form $ax^2 + bx + c$**	At this level, factorising expressions of the form $ax^2 + bx + c$ includes the difference of two squares of the form $ax^2 - c$ where $a > 1$
	• **completing the square**	The technique of completing the square may involve arithmetic using proper or improper fractions without a calculator. Learners may also be asked to interpret the completed square form to state the maximum or minimum value of an expression and the value of x at which this occurs. [See also HA11]
	simplifying expressions involving sums, products and powers, including the laws of indices	

		For example, learners will be expected to • expand $(x-3)(2x+1)(3-5x)$ • factorise $24x^2 - 10x - 6$ • simplify $\frac{9x^2-16}{3x^2-7x+4}$ • express $4x^2 - 12x - 17$ in the form $a(x + b)^2 + c$ where a, b and c are numbers to be found [See also HN2]
HA5.	understand and use standard mathematical formulae; rearrange formulae to change the subject	Given formulae will be more complex at this level and require more steps in order to change the subject.

HA6.	<u>know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments</u> **and proofs**	Learners will need to be able to prove algebraic statements and identities. It is essential that learners understand that, when proving an identity, they should work from the left hand side to the right hand side and not attempt to move terms from the left to the right, as they would when solving an equation. Learners may also be asked to prove simple geometric statements expressed using algebra. In such proofs, learners should give reasons to fully justify statements made. Learners should also be familiar with the type of language used. For example, showing that x satisfies an equation means the learner must use the information given to derive the equation, thereby proving it to be the case. Questions may also be set requiring learners to prove numerical relationships using algebra. For example, • prove $(3x + 2)(5x – 4) – (6x – 3)(x + 2) \equiv 9x^2 – 11x – 2$; or • given a trapezium with perpendicular height $(x + 1)$cm and parallel sides of lengths $(3x + 4)$ cm and $(5x – 2)$ cm, with area 240 cm^2, show that x satisfies the equation $4x^2 + 5x – 239 = 0$ or • given that m and n are even integers, prove that mn is divisible by 4. [See also HA3, HG10, HG25]

HA7.	where appropriate, interpret simple expressions as functions with inputs and outputs**; interpret the reverse process as the 'inverse function';** **interpret the succession of two functions as a 'composite function'**	Function notation, such as $f^{-1}(x)$, $gf(x)$, will be used. Questions may be set requiring learners to find inverse functions of given one-to-one functions. Learners may need to generate a composite function for themselves or complete a composition they have been given to find a rule. The original functions will be relatively simple in all cases. For example, • given the functions $f(x) = x + 3,\quad g(x) = x^2$, defined for all real values of x, find an expression for $gf(x)$. • $f(x) = 2x - 5$ is defined for all real values of x. Find an expression for $f\left(\frac{x}{2}\right)$. A basic understanding of the domain and range of a function is expected. Learners will not be expected to find the range of a composite function. [See also HA4, HA5, HA13]

GRAPHS

Code	Specification Statement	Further clarification or guidance
HA8.	work with coordinates in all four quadrants	
HA9.	plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points, or through one point with a given gradient	For example, learners may be asked to explain whether or not $3x + 4y - 3 = 0$ and $4x - 3y + 3 = 0$ are perpendicular, giving reasons; or to find the equation of the line perpendicular to $2x - 3y + 1 = 0$ which passes through the midpoint of the line joining (3, 4) and (7, –6). Learners will be expected to be familiar with the phrase 'perpendicular bisector'. [See also HA16, HG2]
HA10.	identify and interpret gradients and intercepts of linear functions graphically and algebraically	
HA11.	identify and interpret roots, intercepts, turning points (stationary points) of quadratic functions graphically; deduce roots algebraically, **turning points (stationary points) by completing the square**	For example, find, by completing the square, the turning point of $x^2 + 16x + 62 = 0$. This may be combined with a transformation and so learners may need to deduce the turning point or roots of a related graph from information they have been given or they have derived. [See also HA4, HA13]

HA12.	recognise, sketch and interpret graphs of linear functions, quadratic functions, <u>simple cubic functions, the reciprocal function</u> $y = \frac{1}{x}$, <u>with $x \neq 0$</u>, **exponential functions $y = k^x$ for positive values of k,**	Learners should be able to recognise and draw graphs of exponential functions,, e.g. $y = 3^x$. It is important that they understand how the graphs intersect the y-axis and how they behave for positive and negative values of x. Formal knowledge of asymptotes is not expected, although learners should understand the asymptotic behaviour of such graphs. Learners may be asked to draw and interpret graphs representing exponential growth or decay in order to solve simple problems. [See also HA14, HA24, HR17]
	and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size	Learners should be familiar with the periodic nature and key features of the graphs of these trigonometric functions. These graphs may be used to find the solution to simple trigonometric equations, with or without a calculator, for a given interval. For example, sketch the graph of $y = \sin x$ for $-180° \leq x \leq 180°$ and hence find all the solutions of the equation $\sin x = 0.5$ for $-180° \leq x \leq 180°$. [See also HG20, HG21]

HA13.	**sketch translations and reflections of a given function**	This includes questions involving function notation, e.g. sketch $f(x) = -(x + 4)^3$ using the given sketch of $f(x) = -x^3$. It also includes translations and reflections of the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$. For example, sketch $y = \cos (x + 90°)$. [See also HA7]
HA14.	plot and interpret graphs (<u>including reciprocal graphs</u> **and exponential graphs**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration	Exponential graphs may be used to model real-life situations. An example of exponential decay is when a particle loses ¾ of its mass every second. This can be plotted and interpreted algebraically. Learners may also be required to draw and use graphs representing exponential growth. For example, the number of bacteria, f, in a dish after t minutes is given by $f(t) = 10 \times 4^t$. Draw the graph of $y = f(t)$ for $0 \leq t \leq 2$ and use it to find the time it takes the number of bacteria to double. [See also HA12]

HA15.	**calculate or estimate gradients of graphs**	Gradients of straight lines should be calculated. The gradient of a curve at a point needs to be estimated using a tangent to the curve at that point. Formal differentiation is not in the specification and will not be assessed. [See also HR16]
	and areas under graphs (including quadratic and other non-linear graphs),	The area under a graph may be found either using first principles, splitting the area into trapezia or triangles/rectangles using ordinates, or by application of the trapezium rule.
	and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts	There is an expectation that gradients and areas are interpreted in relation to the graph drawn, e.g. in a velocity-time graph, the tangent at a point gives acceleration, and the area under the graph gives distance travelled.
HA16.	**recognise and use the equation of a circle with centre at the origin;** **find the equation of a tangent to a circle at a given point**	For example, $x^2 + y^2 = 9$ should be recognised as a circle, with centre at the origin and radius 3 units. The tangent to the circle at the point (x_1, y_1) is perpendicular to the radius from the centre to (x_1, y_1). This fact leads to a method for finding the equation of the tangent. [See also HA9, HG6, HG20]

Code	Specification Statement	Further clarification or guidance
HA17.	solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph	Trial and improvement is a numerical not an algebraic method and should not be used.
HA18.	solve quadratic equations of the form $x^2 + bx + c$ and $\boldsymbol{ax^2 + bx + c}$ (**including those that require rearrangement**) algebraically by factorising, **by completing the square and by using the quadratic formula**; find approximate solutions using a graph	The equation may not be given in the form $ax^2 + bx + c = 0$. For example, • solve $(2x + 3)^2 + 3x = 12$, • solve $x = \frac{2}{x+1} + 12$. Learners should be able to solve quadratic equations by any of the stated methods. If a particular method is required, this will be indicated in the question. The technique of completing the square is the basis for the quadratic formula and should help learners to understand why the quadratic formula is valid. The quadratic formula is not given in the examination and must be learned. Learners should state the general formula and show their substitution into it fully before attempting to simplify or use their calculator, if appropriate. For example, solve, by completing the square, $4x^2 – 12x – 17 = 0$. Learners should be aware that trial and improvement is a numerical not an algebraic method and should not be used unless the question indicates that this is appropriate. [See also HA4]

HA19.	solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph	Methods of solving simultaneous equations involving one linear and one quadratic include equating expressions, or substituting the linear equation into the quadratic equation. The linear equation may need rearranging before it is substituted. Often the resulting quadratic may factorise, although this may not always be the case. Learners should be aware that trial and improvement is a numerical not an algebraic method and should not be used unless the question indicates that this is appropriate. [See also HA4, HA18]
HA20.	**find approximate solutions to equations numerically using iteration,**	Learners should understand that not all equations can be solved directly using algebraic processes. Iteration is a numerical method which can be used to find an approximate root of an equation using the output from each step as the input for the next. Subscript notation will be used for iterative formulae and the formula to be used will be given in the question. At this level, the iterative formula may be based on a quadratic equation, even though the roots of such can be found accurately using algebraic methods. Learners need to know that the result of each iteration should be stated to at least 2 decimal places more than the accuracy requested for the final answer. Learners may be required to show that an equation can be rearranged into the form $x = g(x)$ before using the given iterative formula based on this rearrangement. For example, show that $x = 1 + \frac{10}{x+4}$ is a rearrangement of $x^2 + 3x - 14 = 0$, and then use the iteration process, with a starting value $x_1 = 3$ and iterative formula $x_{n+1} = 1 + \frac{10}{x_n + 4}$, to find a root (solution) of $x^2 + 3x - 14 = 0$ correct to one decimal place.

		This leads to … $x_5 = 2{\cdot}5324,\ x_6 = 2{\cdot}5308$. As the values are now consistent in the first two decimal places, the root can confidently be stated as $2{\cdot}5$ to 1 decimal place. Calculators with an ANS key greatly facilitate this process.
	e.g. trial and improvement,	Trial and improvement will require confirmation of solutions, using a half way test, for example, rather than settling on a solution by eye, or by saying ‘closest’.
	decimal search	This is a sign change process used to locate a root of an equation of the form $f(x) = 0$. Learners will be given an appropriate equation and an interval of two consecutive integers between which a root (solution) of the equation lies. They should tabulate values increasing in steps of 0·1 for the first stage of the decimal search. Once this has been done, learners look for a sign change in the values of $f(x)$ and identify the two consecutive values of x with 1 decimal place that produce the sign change. These values then form the new interval to be used in the next stage of the decimal search. The second stage of the decimal search involves repeating the process with steps of 0·01 using the new interval. This continues until the root can be stated to the accuracy required.
	or interval bisection	This is also a sign change process used to locate a root of an equation of the form $f(x) = 0$. Learners will be given an appropriate equation and an interval of two values between which a root (solution) of the equation lies. In a similar way to trial and improvement methods, learners bisect the given interval and evaluate the function at that point. The lower or upper end of the interval is replaced with the midpoint of the interval, depending on whether the sign of the value of $f(x)$ is positive or negative. The process then continues in the same way, narrowing down the interval in which the root lies until the required accuracy is reached.

		A calculator with a TABLE function can greatly facilitate these 3 processes. Good presentation of solutions (for example, by drawing a table) is important to ensure accuracy as it is easy to make slips when using numerical methods.
HA21.	<u>translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution</u>	
HA22.	<u>solve linear inequalities in one</u> **or two** <u>variable</u>(**s), and quadratic inequalities in one variable**; <u>represent the solution set on a number line</u>**, using set notation and on a graph**	At this level, the solution of linear inequalities in one variable may also involve stating the least or greatest integer solution. For example, rearrange the inequality $5n-6>27+3n$ into the form $n>$ a number and write down the least whole number of n that satisfies this inequality. When finding the solutions for quadratic inequalities in one variable, learners may find the critical values (roots) and then make a simple sketch to identify which values are appropriate for the solution of the given inequality. Learners will be expected to identify a range of solutions, clearly knowing which values are included from the interpretation of the inequality sign used. [See also HA12] For example, the solutions of the inequality • $6x-1\le 11$ may be expressed on a number line with a full circle and an arrow to the left of 2 or as the set $\{x: x\le 2\}$ • $x^2+2x-3>0$ may be expressed on a number line by an empty circle at –3 with an arrow to the left and an empty circle at 1 with an

		arrow to the right or as the set $\{x: x < -3 \text{ or } x > 1\}$. When graphing linear inequalities in two variables, the convention of dotted lines for strict inequalities (< and >) is expected. When more than one inequality is graphed, the region satisfying them all (the solution set) should be unshaded and the rejected regions should be shaded. Formal Linear Programming will not be assessed. However, as part of a problem solving question, learners may need to derive simple constraints from information given, draw the graph of these and identify the region representing the solution set. [See also HA12]

SEQUENCES

<table>
<tr><th>Code</th><th>Specification Statement</th><th>Further clarification or guidance</th></tr>
<tr><td>HA23.</td><td>generate terms of a sequence from either a term-to-term or a position-to-term rule</td><td rowspan="2">The process of iteration is to apply a function repeatedly, using the output from one iteration as the input for the next.

A recurrence relation using an iterative formula generates the terms of a sequence and is the formal term-to-term rule. Learners should know the difference between generating a sequence in this way and generating the sequence using the *n*th term (position-to-term rule).

For example, generating a sequence using the recurrence relation $a_1 = 4$ and $a_n = 3a_{n-1} + 4$, leads to

$a_1 = 4$
$a_2 = 3(4) + 4 = 16$,
$a_3 = 3(16) + 4 = 52$, etc.

[See also HA20, HR17]</td></tr>
<tr><td>HA24.</td><td>recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 or a surd) and other sequences</td></tr>
</table>

HA25.	deduce expressions to calculate the *n*th term of linear **and quadratic** sequences	When sequences are presented diagrammatically, it is often easier to identify the *n*th term from the arrangement in the diagram than to use formal algebraic methods examining sequences of numbers. The sequence, therefore, may be found from, for example, spatial arrangements of tiles. When no diagrams are given, methods to find the *n*th term include interpreting a row of constant second differences or forming and solving simultaneous equations. [See also HA2, HA19]

RATIO, PROPORTION AND RATES OF CHANGE

Code	Specification Statement	Further clarification or guidance
HR1.	change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts	
HR2.	understand the concept of density and be able to use the relationship between density, mass and volume; understand the concept of pressure and be able to use the relationship between pressure, force and area	Questions set at this level may involve interpretation and problem solving. Learners will need to apply logic and use their knowledge of pressure in multi-step, unstructured questions.
HR3.	use scale factors, scale diagrams and maps	
HR4.	express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1	
HR5.	use ratio notation, including reduction to simplest form	
HR6.	divide a given quantity into two parts in a given part:part or part:whole ratio; divide a given quantity into more than two parts; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)	
HR7.	express a multiplicative relationship between two quantities as a ratio or a fraction	
HR8.	understand and use proportion as equality of ratios	
HR9.	relate ratios to fractions and to linear functions	

HR10.	define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase / decrease and original value problems, and simple interest including in financial mathematics	
HR11.	solve problems involving direct and inverse proportion, including graphical and algebraic representations	
HR12.	use compound units such as speed, rates of pay, unit pricing, <u>density and pressure</u>	
HR13.	compare lengths, areas and volumes using ratio notation; <u>make links to similarity (including trigonometric ratios)</u> and scale factors	

HR14.	understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **construct and** interpret equations that describe direct and inverse proportion	Equations may be derived from real-life problems involving direct and/or inverse proportion, or from abstract, algebraic variation leading to finding the constant of proportionality. For example, • y is directly proportional to the cube of x, when x = 2, y = 0.32. Find an equation connecting y and x. Leading to $y = \frac{x^3}{25}$. • finding the equation connecting x and y when x = 50 and y = 10 and it is known that y is inversely proportional to x as $y = \frac{k}{x} \rightarrow 10 = \frac{k}{50} \rightarrow k = 500 \rightarrow y = \frac{500}{x}$ or $xy = 500$ Note that when quantities are in direct proportion, the word "direct" may or may not be stated. When quantities are in inverse proportion, this will be indicated.
HR15.	interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion	

HR16.	**interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts**	The gradient at a point on a curve should be found by drawing a tangent at that point. This gives the instantaneous rate of change. Using the average rate of change allows the variable rate of change of a curve to be represented by the constant rate of change between two points on the curve. This, in effect, smooths out the increasing/decreasing rate of change of the curve itself between those two points and is sometimes sufficient interpretation. Average speed is an average rate of change. Understanding this should prevent confusion between finding an average rate of change and an average as a statistical measure. Learners should know that the average rate of change between two points on a curve is found using the chord joining the two points. It is the gradient of the chord and is found using the coordinates of the end points. For example, find the average rate of change of $f(x) = x^2 - 1$ between the points on the curve with x-coordinates 2 and 3. Leading to $\frac{f(3) - f(2)}{3 - 2} = \frac{8 - 3}{1} = 5$. It is important that learners are able to demonstrate understanding that the tangent at a point is the limiting gradient from chords; the shorter the chord, the closer the average rate of change is to the instantaneous rate of change. [See also HA10, HA12, HA15]

HR17.	set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes**	The use of efficient methods is required, such as the use of multipliers and indices in modelling iterative processes. For example, an investment of £1000 at a rate of 3% compound interest each year could be represented by the iterative formula / recurrence relation $V_1 = 1000, \quad V_{n+1} = 1.03 \times V_n$. Learners need to make connections between this formula for finding the value of the investment and the term-to-term relationship between terms of a sequence. This can be used to aid understanding of the formula $\text{Total accrued} = P\left(1+\frac{r}{100}\right)^n$. [See also HA20, HA23, HA24]

GEOMETRY AND MEASURES

PROPERTIES AND CONSTRUCTIONS

Code	Specification Statement	Further clarification or guidance
HG1.	use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description	
HG2.	<u>use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle);</u> <u>use these to construct given figures and solve loci problems;</u> <u>know that the perpendicular distance from a point to a line is the shortest distance to the line</u>	When accurate drawings are asked for, compass constructions are expected. For example, midpoints should be constructed not measured.
HG3.	apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)	
HG4.	derive and apply the properties and definitions of: special types of triangles, quadrilaterals (including square, rectangle, parallelogram, trapezium, kite and rhombus) and other plane figures using appropriate language	

HG5.	use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)	This may be required as part of a formal geometrical proof. A reason should be given for each step and a conclusion stated when the word 'Prove…' is the instruction in the question. [See also HA6]
HG6.	apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs	[See also HG20]
HG7.	identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)	Scale factors could be both negative and fractional. Learners should know that a negative scale factor produces an enlargement of the same magnitude as a positive scale factor but in the opposite direction through the centre. Learners should also know that the enlarged shape is similar to the original shape. [See also HG19]
HG8.	**describe the changes and invariance achieved by combinations of rotations, reflections and translations**	Invariance is specified mathematically by a transformation or combination of transformations that leave some quantities, such as points, lines and shapes, unchanged. For example, • when the line $y = x + 2$ is reflected in the line $x = 1$ the point with coordinates (1, 3) is invariant, • when the shape with coordinates (0, 0), (0, 3), (3, 3), (3, 0) is rotated 180° clockwise about (0, 0) ,reflected in the x-axis and then translated by

		the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ the whole shape is invariant. Rotation, reflection and translation are all isometries of the plane (i.e. they are rigid motions) that result in the final image shape being congruent to the original shape. Transformations can be combined: one transformation followed by another transformation. The resulting transformation can frequently be described by an equivalent single transformation, which learners may be expected to describe. [See also HG19]
HG9.	identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment	[See also HG18]

HG10.	**apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**	These include: angles in the same segment are equal, an angle in a semi-circle is 90°, a radius and tangent meeting at a point are perpendicular, two tangents from a point to a circle are equal in length, an angle at the centre is twice the angle at the circumference, opposite angles of a cyclic quadrilateral add up to 180°, alternate segment theorem, and intersecting chords theorem. Questions may involve proof, working with algebraic quantities. Correct geometrical terminology is expected when learners are asked to justify statements with reasons. When a reflex angle is required, this will be stated in the question. If the word reflex is not included then learners should assume that the angle needed is between 0° and 180°. [See also HA6]
HG11.	solve geometrical problems on coordinate axes	
HG12.	identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres	
HG13.	<u>construct and</u> interpret plans and elevations of 3D shapes	

MENSURATION AND CALCULATION

Code	Specification Statement	Further clarification or guidance
HG14.	use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)	
HG15.	measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings	
HG16.	know and apply formulae to calculate: area of squares, rectangles, triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)	
HG17.	know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; <u>surface area and volume of spheres, pyramids, cones and composite solids</u>	Questions at this level may involve more complex composite solids. They may require a higher level of reasoning and problem solving. Skills may be combined and questions unstructured. [See also HG18, HG19]
HG18.	<u>calculate arc lengths, angles and areas of sectors of circles</u>	At this level, arc lengths and areas may be required for more complex problems. Questions may also be set in context. [See also HN8, HG9, HG17]

<table>
<tr>
<td>HG19.</td>
<td>apply the concepts of congruence and similarity, including the relationships between lengths, areas and volumes in similar figures</td>
<td>Learners need to understand the relationship between length, area and volume scale factors for similar figures, i.e.

<table>
<tr><td>Dimension</td><td>1D (length)</td><td>2D (area)</td><td>3D (volume)</td></tr>
<tr><td>Scale factor</td><td>s</td><td>s^2</td><td>s^3</td></tr>
</table>

Problems may involve reverse situations. For example, learners may be given the area of the smaller of two similar figures and need to find a length of the larger figure.

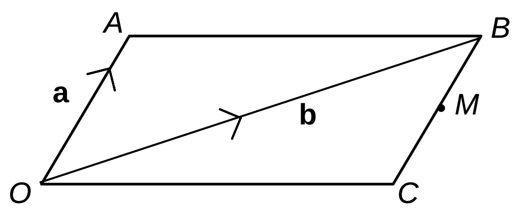
[See also HG7]</td>
</tr>
<tr>
<td>HG20.</td>
<td>know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$,
$\tan\theta = \frac{opposite}{adjacent}$,
apply them to find angles and lengths in right-angled triangles in two dimensional figures and, where possible, general triangles in two and three dimensional figures</td>
<td>For example, Pythagoras' theorem may be used to find the distance between two points on a coordinate grid. At this level a supporting diagram may not be given and learners may be expected to find the distance from coordinates only, generating their own diagram. Pythagoras' theorem can also be quoted in 3 dimensional form. Although it is possible to avoid this by applying the 2 dimensional form twice, using the 3 dimensional form is less prone to errors and should be encouraged where possible. For example, learners may be asked to find the maximum length across a cuboid with length, height and width x, y and z. This would be found using $\sqrt{x^2 + y^2 + z^2}$.

Pythagoras' theorem and trigonometry may both be required to solve multi-step, unstructured problems.

For trigonometric questions, learners should be careful to make sure that their calculators, if appropriate, are in degrees. It is possible that questions will be set where learners are not permitted to use a calculator.</td>
</tr>
</table>

		Problems may be in context and learners may need to apply knowledge of bearings/compass points (N, NE, E, SE, S, SW, W, NW), angles of elevation and/or depression . Learners may need to apply symmetry and general knowledge about triangles, quadrilaterals and regular shapes, for example, to solve problems not set in a context. For example, using the symmetry on an isosceles triangle to create two right-angled triangles. Other multi-stage problems may include a triangle that can be split into one or two right-angled triangles of differing sizes. For example, where angles of elevation from one point (middle of a road) in different directions (tops of the buildings directly on either side of the road), lead to 2 right-angled triangles; or angles of elevation from one point (a boat) to two different points (the top of a cliff and the top of a flag pole which stands on top of the cliff). In 3 dimensions right-angled triangles need to be identified within standard 3D figures, such as pyramids, cuboids and prisms. 3D coordinate grids will not be used. [See also HG6, HG15, HG21]
HG21.	know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0°, 30°, 45°, 60°$ and $90°$; know the exact value of $\tan\theta$ for $\theta = 0°, 30°, 45°$ and $60°$	[See also HG20, HG22]

HG22.	**know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule,** **$a^2 = b^2 + c^2 - 2bc\cos A$, to find unknown lengths and angles**	The ambiguous case of the sine rule will not be assessed. Learners will need to have a clear understanding of the naming conventions of sides and vertices of a triangle for these rules to make sense. [See also HG1] HG22 and HG23 can include multi-step problems or reverse problems, e.g. given the lengths of all 3 sides of a triangle, find the area, which involves cosine rule to find an angle and then ½$ab\sin C$ to find the area. Learners need to be clear about which rule to apply and when as they may have to determine this for themselves. It may be helpful if they know that the cosine rule and ½$ab\sin C$ can be thought of as Side-Angle-Side (SAS) formulae.
HG23.	**know and apply Area $= \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle**	

Code	Specification Statement	Further clarification or guidance
HG24	describe translations as 2D vectors	
HG25	apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **use vectors to construct geometric arguments and proofs**	Vectors will be printed as, for example, **AB** or **a.** In relation to vectors, learners will need to use, understand and interpret • the midpoint of a line and a line divided using a given ratio • parallel vectors as scalar multiples of each other • equal vectors • opposite direction of vectors. It is important that the difference between parallel and collinear is understood in the interpretation of findings. Formal application of the ratio theorem is not expected. For example, the diagram shows a parallelogram *OABC*. 

		OA = **a** and **OB** = **b.** M is the midpoint of BC. Find the vector **OM** and the vector **AB** in terms of **a** and **b**. The point *N* lies on AB and is such that *AN* : *NB* = 1 : 4. Show that **ON** = $\frac{4}{5}\mathbf{a}+\frac{1}{5}\mathbf{b}$. [See also HA6]

PROBABILITY

Code	Specification Statement	Further clarification or guidance
HP1.	record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees	
HP2.	apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments	
HP3.	relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale	
HP4.	apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one	
HP5.	understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size	
HP6.	enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams	
HP7.	construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities	
HP8.	calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions	

HP9.	**calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams**	On occasion, learners may be required to complete a given diagram, tree or table. If this is not the case, learners should then draw their own suitable diagram, tree or table, as appropriate. Diagrams are very helpful in interpreting otherwise complex information. When a diagram, tree or table outline is given with the instruction 'You may use the diagram to help you', learners are strongly advised to use it. Problem solving questions may include non-replacement situations. A particular method may be required, although some examination questions will leave decision-making, regarding method, to the learner. Set and probability notation associated with Venn diagrams is required, e.g. $P((A \cup B') \cap C) = 0{\cdot}06$. Venn diagrams may represent more than 2 sets.

STATISTICS

Code	Specification Statement	Further clarification or guidance
HS1.	infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling	Formal sampling methods such as stratified sampling, quota sampling and so on are not required. It is expected that learners will have an understanding of the notion of simple *random* sampling. Questions may be more complex and multi-step at this level
HS2.	designing and criticising questions for a questionnaire, including notion of fairness.	
HS3.	interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, <u>tables and line graphs for time series data</u> and know their appropriate use	
HS4.	**construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**	It will be important to understand skewness when interpreting grouped data. Interpretation will include finding the median and other percentile outcomes. Learners should know that, in a histogram with unequal class widths, the area is proportional to the frequency of the class. It is also important that, for a histogram of this type, representing continuous grouped data, there are no gaps between the bars, the axes are both labelled (the vertical axis being frequency density) and the scale used for the vertical axis starts at 0 and the scale used for both axes is uniform. [See also HS5]

HS5.	interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data, **including box plots** • appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers**, quartiles and inter-quartile range**)	Box plots show summary statistics and are an alternative representation to cumulative frequency graphs. Similar expectations of interpretation will be required. Learners are expected to create box plots from cumulative frequency diagrams. A single question may involve both types of presentation and may include comparisons of a distribution represented by a cumulative frequency graph with that represented by a box plot. Learners should have an understanding of the advantage of using the inter-quartile range rather than the range. [See also HS4]
HS6.	apply statistics to describe a population	
HS7.	use and interpret scatter graphs of bivariate data; recognise correlation <u>and know that it does not indicate causation;</u> <u>draw estimated lines of best fit;</u> <u>make predictions;</u> <u>interpolate and extrapolate apparent trends whilst knowing the dangers of so doing</u>	

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2. ASSESSMENT OBJECTIVES

The assessment objectives for this new GCSE Mathematics specification are shown below. These assessment objectives have changed significantly from those within the 2010 Linear specification. The new assessment objectives give a greater emphasis to problem solving, reasoning, interpreting and communicating.

Each of the assessment objectives has been broken down into strands (shown below), to add clarity to the main statements.

The weightings that apply to the assessment objectives (shown below) are no longer the same across the Foundation and Higher tiers.

Assessment Objectives		Weighting	
		Higher	Foundation
AO1	**Use and apply standard techniques** Learners should be able to: • accurately recall facts, terminology and definitions • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions	40%	50%
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • make deductions, inferences and draw conclusions from mathematical information • construct chains of reasoning to achieve a given result • interpret and communicate information accurately • present arguments and proofs • assess the validity of an argument and critically evaluate a given way of presenting information Where problems require learners to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding assessment objective.	30%	25%
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes • make and use connections between different parts of mathematics • interpret results in the context of the given problem • evaluate methods used and results obtained • evaluate solutions to identify how they may have been affected by assumptions made Where problems require learners to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding assessment objective.	30%	25%

The new Assessment Objectives are broken down into strands and elements, as shown below. Questions set in examination papers will target one or more of the elements.

Some of the strands have just a single element, e.g. see elements 1.1 and 1.2 below.

In the Sample Assessment Materials (SAMs), the elements targeted by each question are referenced in the mark schemes.

AO1 Use and apply standard techniques			
Strands		**Elements**	
1	Accurately recall facts, terminology and definitions	**1.1**	Accurately recall facts, terminology and definitions
2	Use and interpret notation correctly	**1.2**	Use and interpret notation correctly
3	Accurately carry out routine procedures or set tasks requiring multi-step solutions	**1.3a**	Accurately carry out routine procedures
		1.3b	Accurately carry out set tasks requiring multi-step solutions

At least 80% of the AO1 marks will target strand 3.
Generally, there will be more marks targeting element 1.3a than 1.3b at the Foundation tier and more marks targeting 1.3b than 1.3a at the Higher tier.

AO2 Reason, interpret and communicate mathematically			
Strands		**Elements**	
1	Make deductions, inferences and draw conclusions from mathematical information	**2.1a**	Make deductions to draw conclusions from mathematical information
		2.1b	Make inferences to draw conclusions from mathematical information
2	Construct chains of reasoning to achieve a given result	**2.2**	Construct chains of reasoning to achieve a given result
3	Interpret and communicate information accurately	**2.3a**	Interpret information accurately
		2.3b	Communicate information accurately
4	Present arguments and proofs	**2.4a**	Present arguments
		2.4b	Present proofs
5	Assess the validity of an argument and critically evaluate a given way of presenting information	**2.5a**	Assess the validity of an argument
		2.5b	Critically evaluate a given way of presenting information

Element 2.4b, Presenting proofs, is a requirement at Higher tier only.

AO3 Solve problems within mathematics and in other contexts			
Strands		**Elements**	
1	Translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes	**3.1a**	Translate problems in mathematical contexts into a process
		3.1b	Translate problems in mathematical contexts into a series of processes
		3.1c	Translate problems in non-mathematical contexts into a *mathematical* process
		3.1d	Translate problems in non-mathematical contexts into a series of *mathematical* processes
2	Make and use connections between different parts of mathematics	**3.2**	Make and use connections between different parts of mathematics
3	Interpret results in the context of the given problem	**3.3**	Interpret results in the context of the given problem
4	Evaluate methods used and results obtained	**3.4a**	Evaluate methods used
		3.4b	Evaluate results obtained
5	Evaluate solutions to identify how they may have been affected by assumptions made	**3.5**	Evaluate solutions to identify how they may have been affected by assumptions made

There will be a greater emphasis on strands 1, 2 and 3 than strands 4 and 5.
Within strand 1, there will be a greater emphasis on multi-step tasks requiring a series of processes (elements 1b and 1d) than tasks that require a single process (elements 1a and 1c).

EXAMPLES

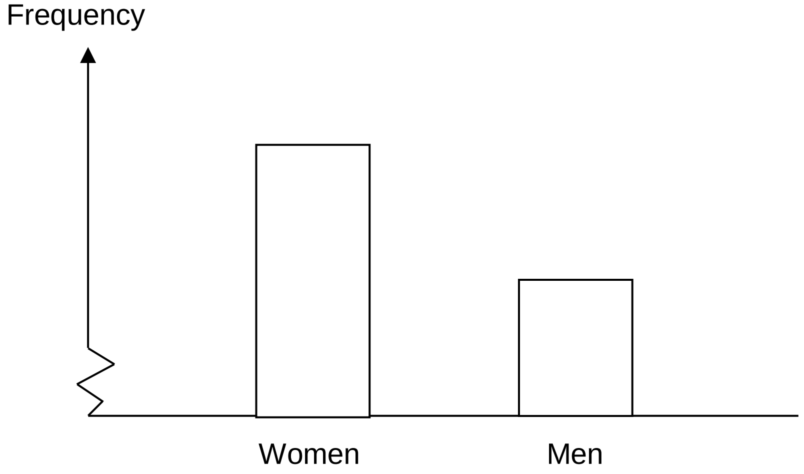
These examples are taken from the Sample Assessment Materials. This is a selection of questions from these materials that assess elements of AO2 and AO3. They do not represent every type of AO2 or AO3 question that could be asked.

SAMS COMPONENT 1 FOUNDATION Q18

18. *(a)* Explain why the statements that accompany each of the following diagrams in a newspaper may not be true.

Your comments should be based on the diagrams and not on your personal opinion.

(i) Taken from an item about accidents in the home. [1]



'Twice as many women as men have accidents in the home.'

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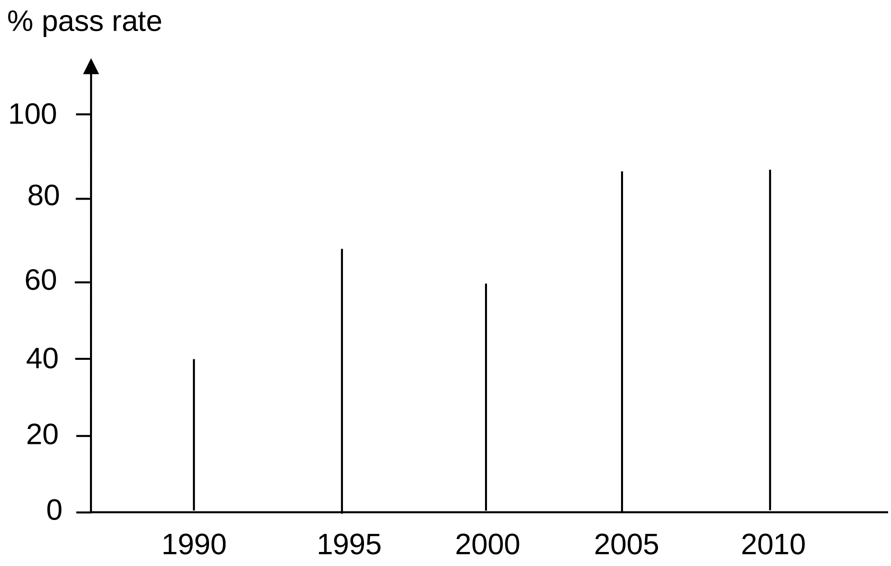
Parts (a)(i) and (ii) are examples of AO2 questions, where the candidate has to critically evaluate given ways of presenting information (2.5b).

(ii) Taken from an item about a school's examination percentage pass rates. [1]



'The percentage pass rate has remained constant between 2005 and 2010'

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(b) Is the following statement true or false?
You must give a full explanation for your decision. [1]

'Every whole number that ends in a 3 is a prime number'.

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This is an example of a one-mark question where the candidate has to assess the validity of an argument (2.5a).

SAMS COMPONENT 1 HIGHER Q12 & FOUNDATION Q29

12. A building company used 24 workers to prepare a building site.
The site measured 30 acres and the work was completed in 10 days.

(a) The company is asked to prepare another site measuring 45 acres.
This work has to be completed in 15 days.
Calculate the least number of workers the company should employ for this work. [3]

This is an example of a predominantly AO3 question, where the candidate has to translate the problem set in a non-mathematical context into a mathematical process (3.1c).

(b) State one assumption you have made in your answer to part *(a)*.
How would your answer to part *(a)* change if you did not make this assumption? [2]

This is an example of an AO3 question, where the candidate is required to evaluate the method used, by stating an assumption that has been made (3.4a). The candidate then has to evaluate the solution, by identifying how it may be affected by the assumption made; in this case, considering the effect of the assumption not being made (3.5).

21. The diagram below shows a composite shape formed by joining two rectangles.

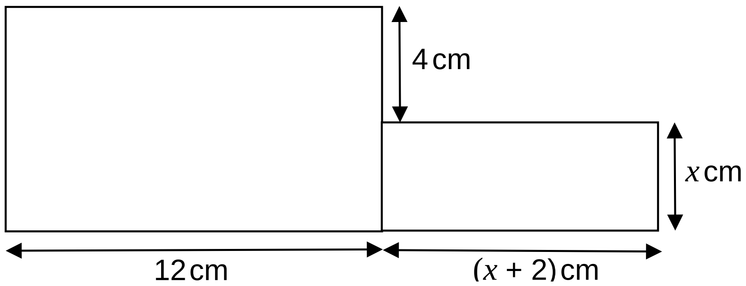

Diagram not drawn to scale

The area of the larger rectangle is 4 times the area of the smaller rectangle.

Calculate the dimensions of the smaller rectangle.
You must justify any decisions that you make. [7]

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This is an example of a question that targets AO3, where the candidate has to evaluate the results obtained (3.4b); in this case, by reflecting on the solutions obtained, discarding the negative root, and justifying why they have discarded it. The candidate then has to interpret the result in the context of the problem (3.3), by giving the dimensions of the smaller rectangle.

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17. The inside of a large industrial container has a height of 3 metres, measured correct to the nearest 10 cm.

It is used to hold a single stack of flat metal plates.
Each metal plate has a thickness of 4 centimetres, measured correct to the nearest millimetre.

(a) Find the greatest possible number of these plates that could be stacked in the container.

[3]

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Part (a) is an example of a predominantly AO3 question, where the candidate has to translate the problem set in a non-mathematical context into a series of processes (3.1d). In part (b), the candidate is required to present a reasoned argument (2.4a).

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(b) Damian states that it may not be possible to stack 73 of these plates in the container.
Show that Damian is correct.

[3]

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19. A cylinder is made of bendable plastic.

A dog's toy is made by bending the cylinder to form a ring.

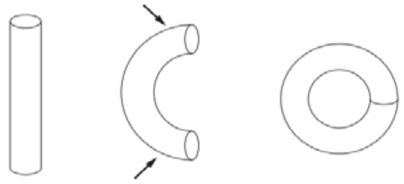

Diagram not drawn to scale

The inner radius of the dog's toy is 8 cm.
The outer radius of the dog's toy is 9 cm.

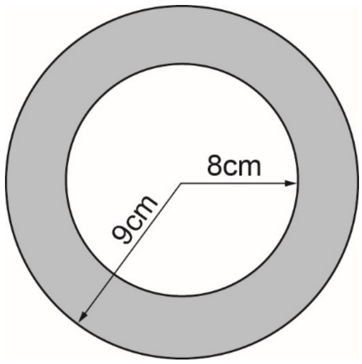

Diagram not drawn to scale

Calculate an approximate value for the volume of the dog's toy.
State and justify what assumptions you have made in your calculations and the impact they have had on your results.

[7]

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This is an example of an AO3 question, where the candidate has to:

- translate a problem set in a non-mathematical context into a series of mathematical processes (3.1d);
- evaluate the method used by making assumptions in order to answer the question (3.4a);
- evaluate the solution by identifying how it may have been affected by the assumptions made (3.5).

20. A team of examiners has 48 000 examination papers to mark.

It takes each examiner 1 hour to mark approximately 16 papers.

(a) The chief examiner says that a team of 25 examiners could mark all 48 000 papers in 8 days.
What assumption has the chief examiner made?
You must show all your calculations to support your answer. [4]

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(b) Why is the chief examiner's assumption unrealistic?
What effect will this have on the number of days the marking will take? [2]

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This is an example of an AO3 question, where the candidate is asked to evaluate the results obtained, by criticising the assumption made (3.4b). The candidate then has to evaluate the solution, by identifying how it may be affected by the assumption (3.5).

3. LINKS TO TEACHING AND LEARNING RESOURCES

WJEC Eduqas is constantly creating free teaching and learning resources to support the specifications. Follow the links below to access the support available:

Specification
http://www.eduqas.co.uk/qualifications/mathematics/gcse/

Sample assessment materials
http://www.eduqas.co.uk/qualifications/mathematics/gcse/

Digital
resources http://resources.eduqas.co.uk/Pages/ResourceByArgs.aspx?subId=38&lvlId=0

Question bank
http://www.wjec.co.uk/question-bank/

Online Exam Review
http://oer.wjec.co.uk/