Chapter 4: AC Circuits and Passive Filters

Learning Objectives:

At the end of this topic you will be able to:

- use V-t, I-t and P-t graphs for resistive loads
- describe the relationship between rms and peak values for sinusoidal signals
- select and apply the following equations:
  - capacitive reactance $X_C = \frac{1}{2\pi fC}$
  - inductive reactance $X_L = 2\pi fL$
  - series impedance $Z = \sqrt{R^2 + X^2}$
- draw, recognise and interpret the output signals of RC passive filters using linear-log and log-log output graphs and describe the advantage of buffering passive filters
- recognise, analyse, design and draw circuits for high-pass and low-pass passive RC filters and passive LC band-pass filters
- select and apply the following equation for high-pass and low-pass passive RC filters:
  - break frequency $f_b = \frac{1}{2\pi RC}$
- select and apply the following equations:
  - for a parallel LC network, resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$
  - dynamic resistance of a resonant circuit $R_D = \frac{L}{r_L C}$
  - Q-factor $Q = \frac{f_0}{\text{bandwidth}} = \frac{2\pi f_0 L}{r_L}$

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Properties of audio signals

Normally, the human ear can detect audio frequencies ranging from about 20 Hz to 20 kHz, though as people get older, their range of hearing decreases, particularly at the high frequency end. Human hearing is most sensitive to sounds in the middle range, from about 200 Hz to 2 kHz, corresponding with the range of frequencies found in speech.

The sounds produced by a human voice or musical instrument have three characteristics:

Pitch:
- indicates the frequency of the sound

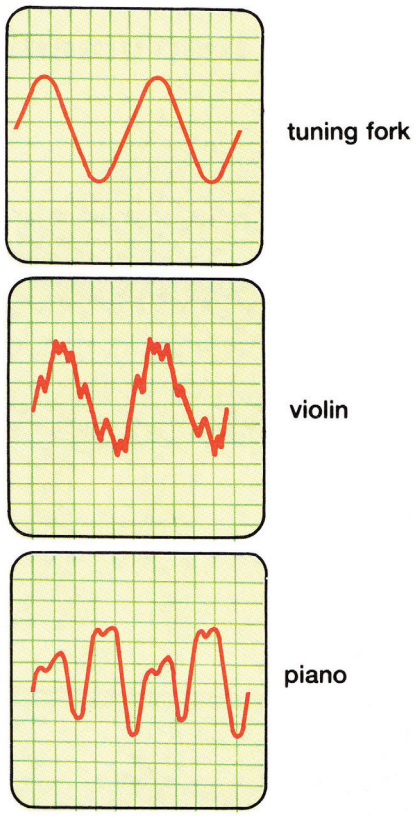
Loudness:
- indicates the amplitude of the sound - the larger the amplitude the more energy the sound wave contains and hence the louder the sound

Tone:
- describes the 'quality' of the sound
- depends on the shape of the waveform - determined by the mixture of frequencies that it contains. Two notes may have the same pitch and loudness, but do not sound the same because they have different mixtures of frequencies and so different waveforms.

Introduction to Filters

Filters are used extensively in audio and communication systems. They allow some signal frequencies to pass through while blocking others.

The pictures on the right show oscilloscope traces for the same note, played on different musical instruments.



Even though they have the same loudness, the notes sound different because the instruments do not produce **pure** notes (containing just one frequency.)

Rather, the sounds consist of a principal frequency and a series of weaker frequencies. The principal, or fundamental note is also referred to as the **first harmonic**.

Suppose that the first harmonic has a frequency **f**, then:

- the components with a frequency **2f** is called the **second harmonic**
- the component with frequency **3f** is called the **third harmonic** and so on.

The harmonic content defines the shape of the waveform and the tone of the sound. Different instruments produce notes with a different combination of harmonics.

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Investigation 4.1:

To appreciate this, we consider two very different but simple-looking waves, the sine wave and the square wave:

- connect a function generator to a loudspeaker set to produce a 1 kHz sine wave
- adjust the amplitude so that the sound produced can be heard comfortably
- change the output from sine wave to a square wave. Notice the difference in the sound produced by the two signals
- complete the first row of the table
- change the frequency to each of the frequencies listed in turn and repeat the procedure. You may need to switch between sine wave and square wave several times before deciding if there is any difference.

Frequency / kHz	Comment on any difference in what you hear.
1	
2	
3	
4	
5	
6	
7	
8	

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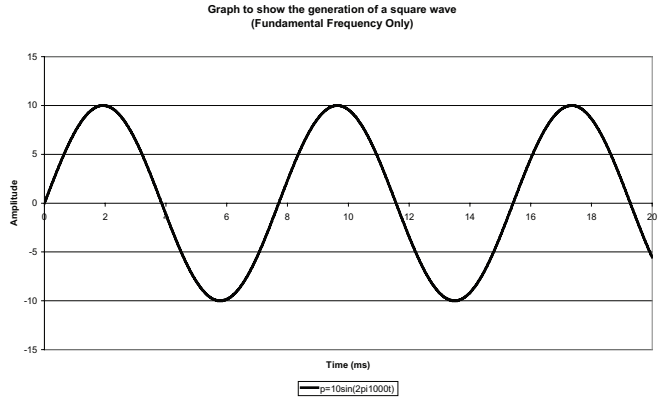
Analysis of a square wave

Completing the table in investigation 1 should have revealed a strange result.
At around 6 kHz, the sine wave and square wave sound the same.

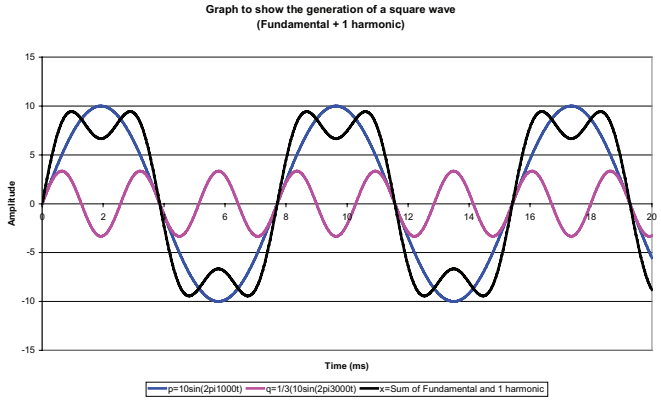
In the early 19th century, a mathematician, Jean Baptiste Fourier, developed a theory that any waveform, no matter how complex, can be made up of a series of sine waves having different amplitudes and frequencies.

Fourier analysis of a square wave with **equal** mark/space ratio reveals that it is made up of the fundamental frequency of the square wave, combined with an infinite number of odd harmonics - sine wave signals of ever-decreasing amplitude, with frequencies that are odd multiples of the fundamental.
The following diagrams illustrate what happens as we keep adding these harmonics:

Firstly – just the fundamental

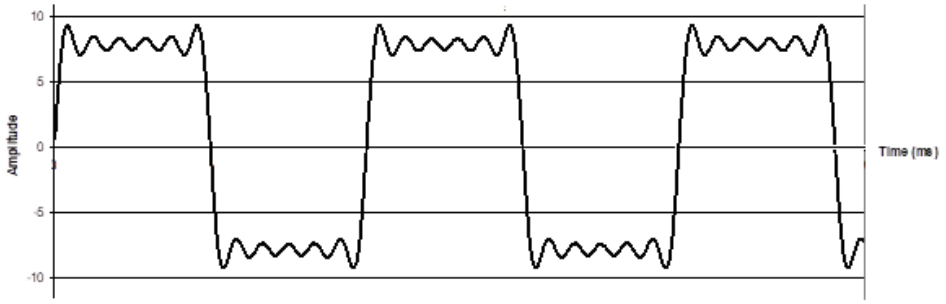


Next, adding the third harmonic.



Already, the resulting waveform is starting to look slightly more ‘square’.

Adding more odd harmonics, with diminishing amplitudes, makes the waveform look even more like a square wave. The final diagram shows the result of adding the 3rd, 5th , 7th, 9th and 11th harmonics.



The addition of five harmonics is still a long way off the ‘perfect’ square wave. Fourier’s theorem states that to create the perfect square wave requires an **infinite** number of odd harmonics.

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Returning to the question of why a 6 kHz sine wave sounds the same as a 6 kHz square wave, the square wave consists of the fundamental frequency, a 6 kHz sine wave, and an infinite number of 'odd' harmonics. The first of these has a frequency of 18 kHz (3 x 6 kHz) followed by a harmonics at 30 kHz (5 x 6 kHz), 42 kHz (7 x 6 kHz) etc.

However, the harmonic, at 18 kHz, is already approaching the limit of what humans can hear (20 Hz to 20 kHz). The majority of the population hear just the fundamental, a sine wave of frequency 6 kHz. It is not surprising that it sounds the same as the 6 kHz sine wave.

Similarly, at a frequency of 7 kHz, there is no discernable difference as the third harmonic, at a frequency of 21 kHz (3 x 7 kHz) is outside of the range of human hearing.

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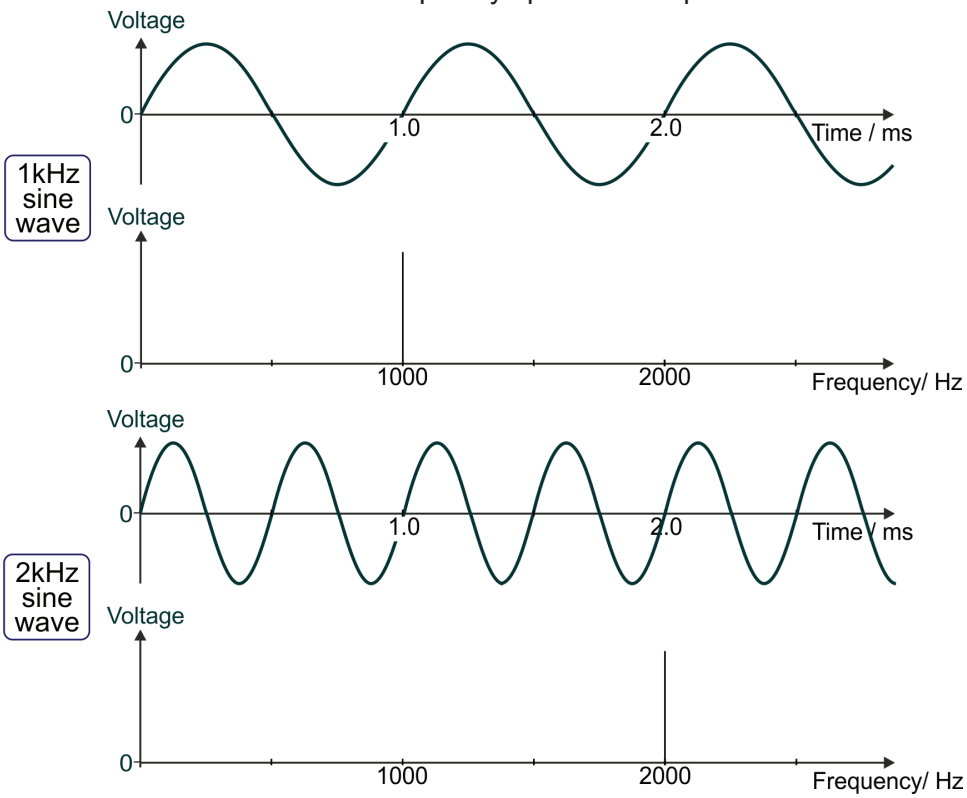
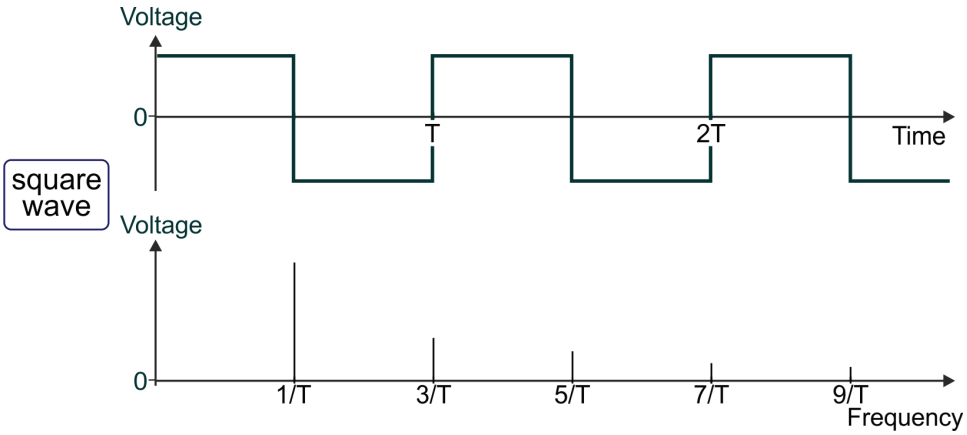
Frequency Spectrum

We briefly considered frequency spectrum graphs in the earlier chapter on op-amps. We now consider the frequency spectrum graph for a sine wave, a square wave and audio signals.

The diagrams below show the waveform and frequency spectrum for pure sine waves of frequency 1 kHz and 2 kHz.



The next diagrams show the waveform and frequency spectrum for a square wave, having a periodic time **T**. Only the fundamental and first four harmonics are shown in the frequency spectrum.



In communication systems, all signals can be analysed in this way to determine their frequency content. Doing so allows us to determine the **bandwidth** needed to transmit the signal.

The bandwidth is the **range** of frequencies required to make up the signal, i.e. the highest frequency component minus the lowest frequency component.

For a sine wave, the bandwidth is zero, since there is only one frequency present in the signal. In the same way, the bandwidth of a square wave is infinite, as it contains an infinite series of sine waves.

Every other signal lies between these extremes. Sometimes the signal bandwidth is deliberately reduced because the effect on quality of communication is marginal or to reduce costs.

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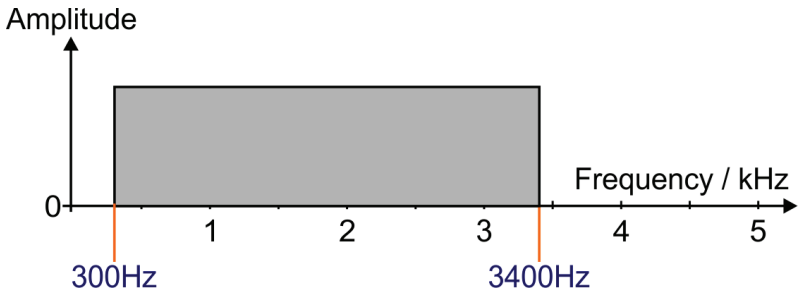
Bandwidth requirement for transmitting speech

During normal conversation the human voice produces constantly changing ranges of amplitude (loudness) and frequency (pitch). To manage this, communication systems assume a maximum amplitude and a maximum bandwidth.

For example, the number of separate telephone channels that can be transmitted over a telephone network depends on the bandwidth allocated to each channel. This bandwidth should be sufficient to identify the caller's voice and to understand what they are saying.

Rather than allocate a bandwidth from 20 Hz to 20 kHz, to cover the full audio frequency range, the public telephone system in the UK uses a more economical bandwidth of 3.1 kHz, allowing frequencies between 300 Hz to 3.4 kHz and filtering out the other, unnecessary, frequencies.

This is usually represented by a rectangle, as shown in the following diagram.

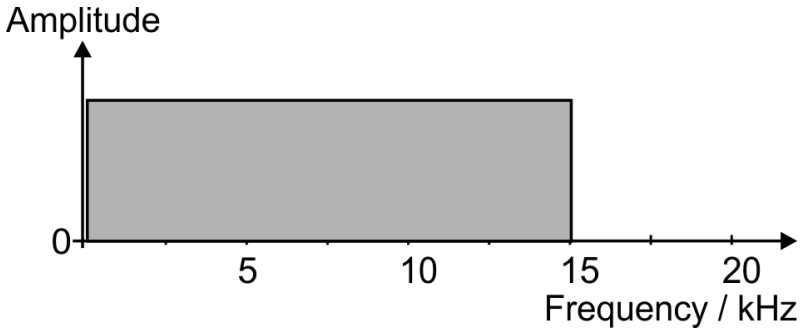


Bandwidth requirement for transmitting music

In general, musical instruments produce sounds with a bandwidth covering the full audio range, i.e. about 20 Hz to about 20 kHz. This is significantly greater than the 3.1 kHz bandwidth limit of the telephone system so it is no wonder that music transmitted over the telephone is of poor quality.

In practice, to reduce costs, broadcasters of music restrict the frequency range to 30 Hz to 15 kHz approximately.

Using the same approach as for speech, the frequency spectrum of a music signal is represented by the following diagram:



Filter circuits are used to control the bandwidth allocated to speech and music signals in communication systems.

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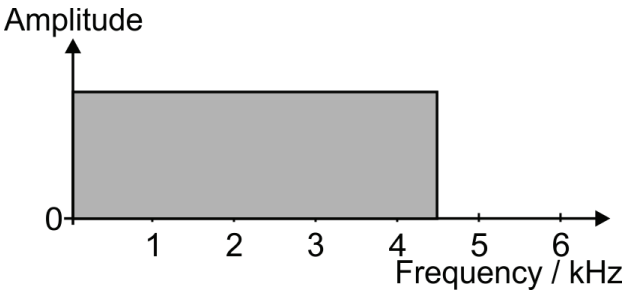
Ideal Filters

Filters fall into three main categories:

- Low-pass Filter (LPF)
- High-pass Filter (HPF)
- Band-pass Filter (BPF).

The low-pass filter

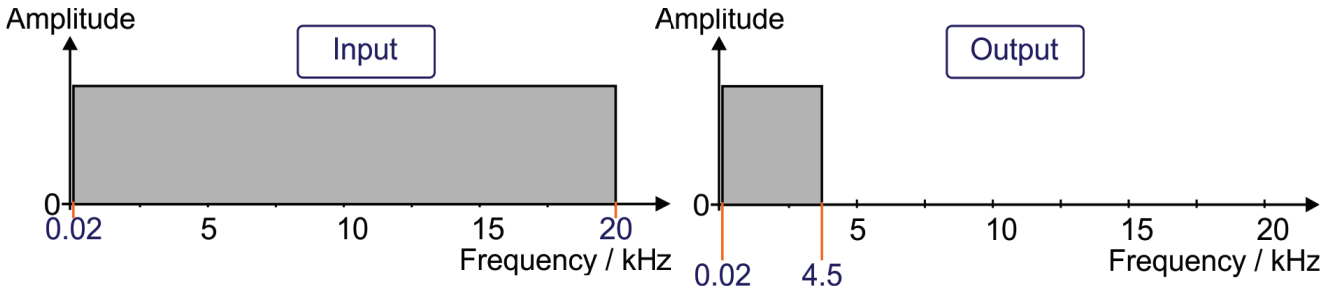
As its name suggests, this type of filter allows low frequency signals to pass through unaffected, but blocks high frequency signals. Its frequency spectrum is shown in the next diagram, drawn for a LPF which blocks frequencies above 4.5 kHz.



This filter passes all frequencies below 4.5 kHz without any change but blocks higher frequencies, so that no trace of them appears at the output.

Example 1:

The graphs show the effect of this low-pass filter on a speech signal.

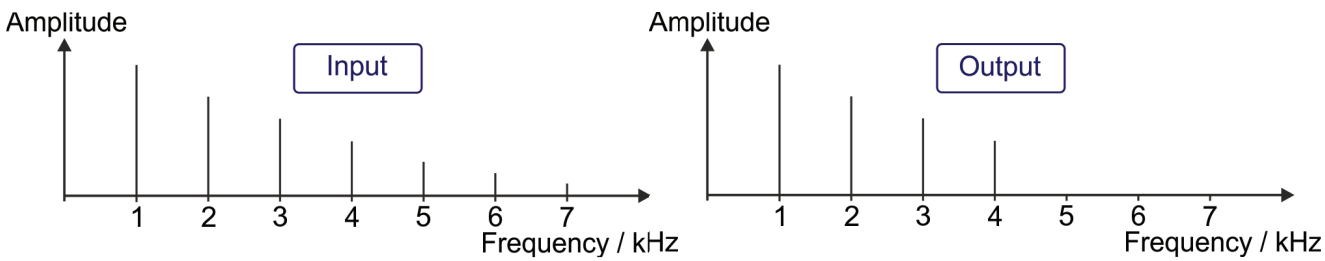


Note:

Even though the bandwidth has been severely cut back, the signal will still be distinguishable. It is the quality that has been lost by cutting off the high frequency components.

Example 2:

The graphs show the effect of this low-pass filter on the frequency spectrum of a square wave signal containing frequency components above and below 4.5 kHz.



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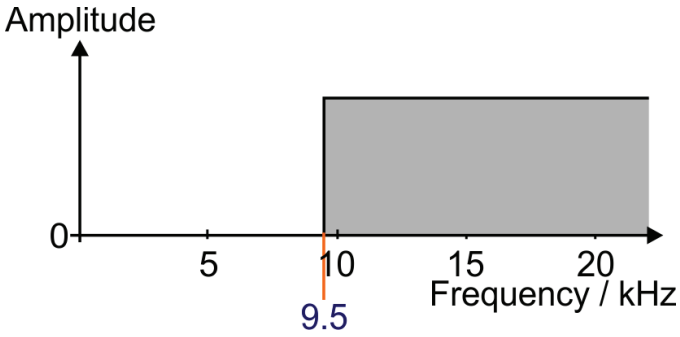
The high-pass filter

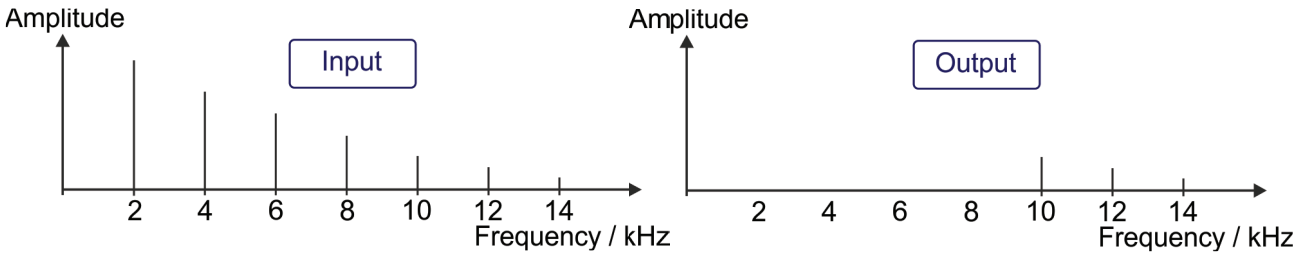
This type of filter allows high frequency signals to pass through unaffected, but blocks high frequency signals. Its frequency spectrum is shown in the next diagram, drawn for a HPF which blocks frequencies below 9.5 kHz.



For this filter all frequencies above 9.5 kHz would be allowed through without any changes. All frequencies below 9.5 kHz would be blocked and no trace of them would appear at the output.

Example 1:

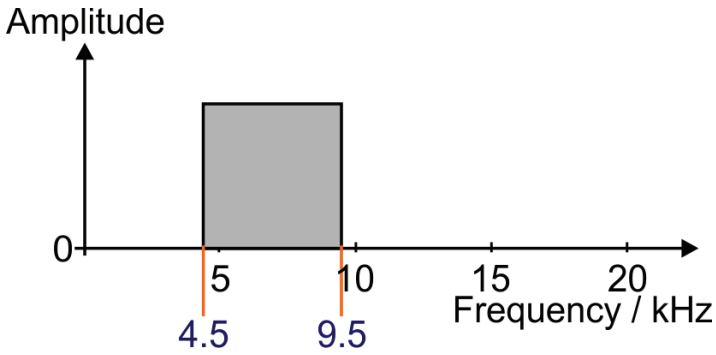
The frequency spectrum diagram of a complex signals before and after passing through the 9.5 kHz HPF filter is:



The band-pass filter

This filter allows only a certain range or ‘band’ of frequencies to pass through unaffected. Any signal having a frequency either lower or higher than this band is blocked.

For the filter described in the following diagram, all frequencies between 4.5 kHz and 9.5 kHz are allowed through unaffected. All frequencies below 4.5 kHz or above 9.5 kHz are blocked and no trace of them appears at the output.



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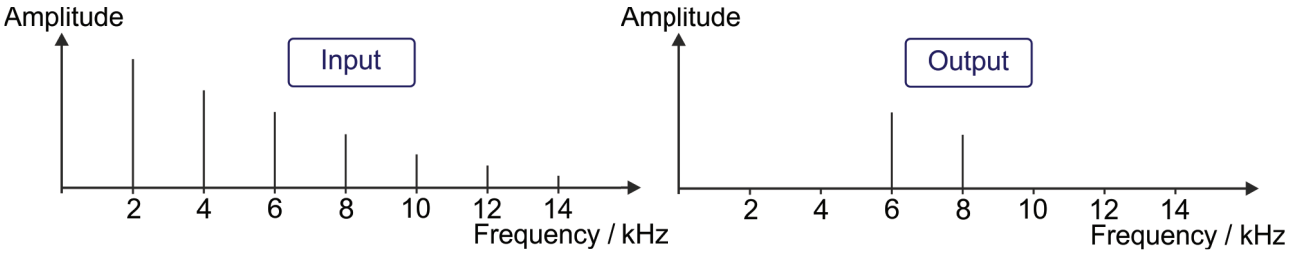
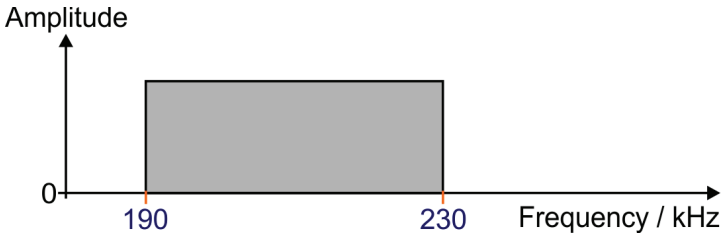
Example 1:

The frequency spectrum diagram of a complex signals before and after passing through the BPF filter is:

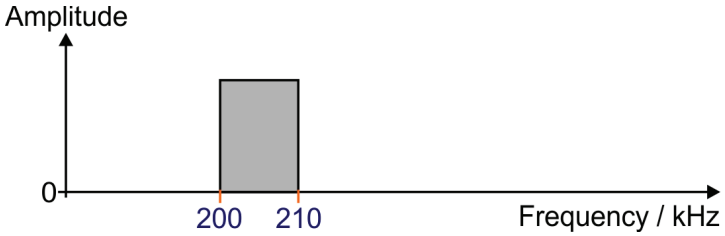


Example 2:

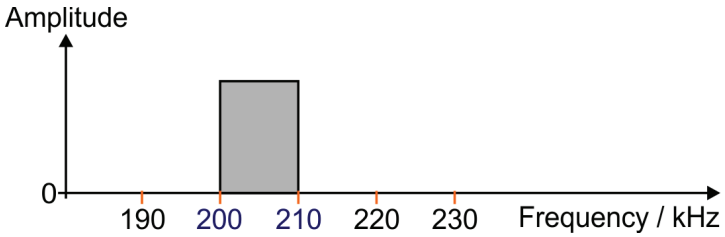
A radio signal has the following frequency spectrum.



It is applied to the input of a band-pass filter, which has the following characteristic.



The resulting signal at the output of the filter is:



These examples assume that the filters have **ideal** characteristics, i.e. perform exactly as we would like them to do. In practice, when we try to build such filters, we do not obtain ideal characteristics but can get very close.

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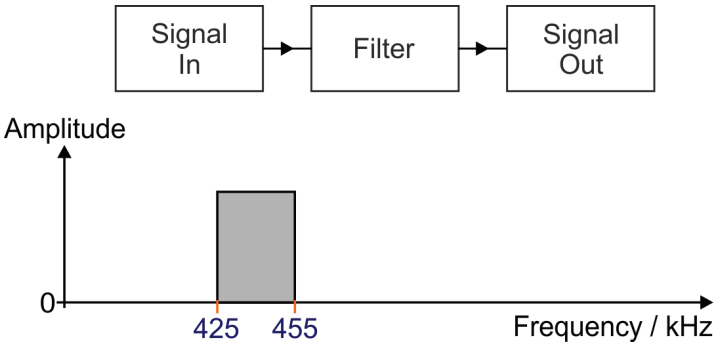
Exercise 4.1

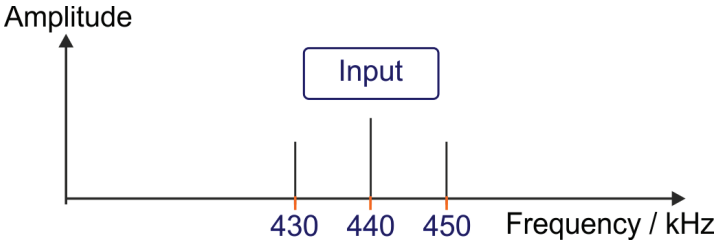
An engineer is testing a filter. The following diagrams show the block diagram of the test system and the frequency characteristics of the filter.



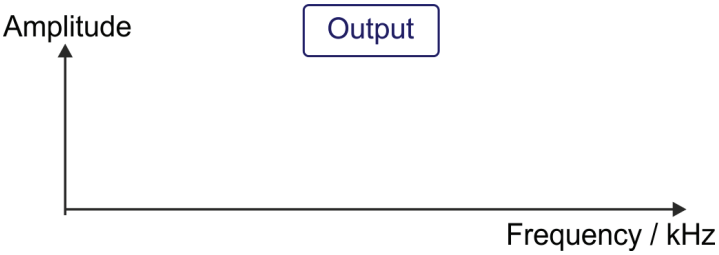
(a) What is the name of this type of filter?

……

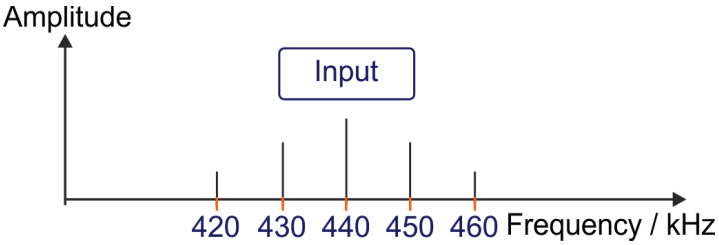
(b) The engineer inputs a radio signal having the following frequency spectrum.



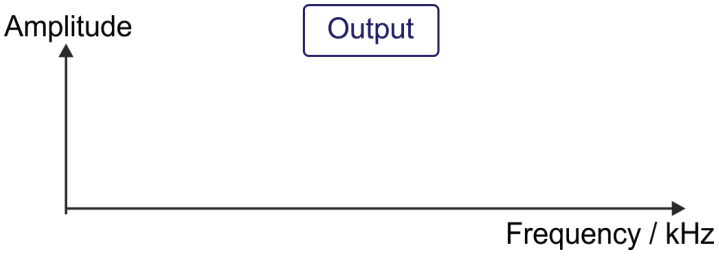
Sketch the frequency spectrum of the resulting output signal, labelling relevant frequencies.



(c) The engineer then inputs a different signal, having the following frequency spectrum.



Sketch the frequency spectrum of the resulting output signal, labelling relevant frequencies.



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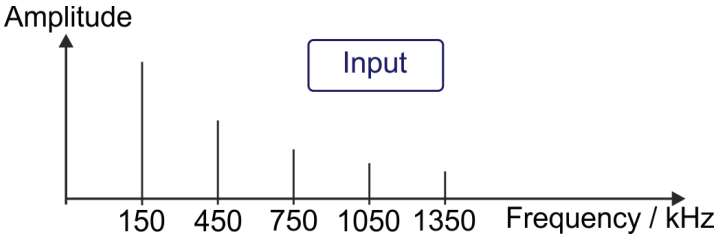
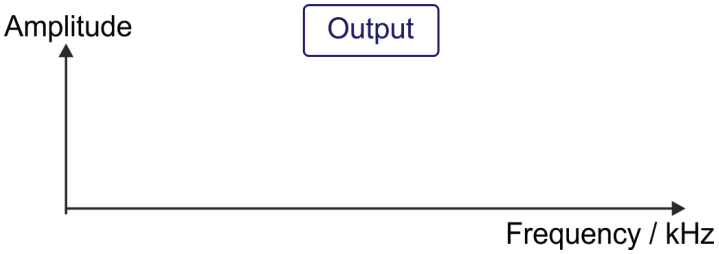
(d) Finally, the engineer inputs a square wave signal with the following frequency spectrum.



Sketch the frequency spectrum of the resulting output signal, labelling relevant frequencies.



Real Filters

Filters fall into two main categories:

- passive filters
- active filters.

This section examines only passive filters. Active filters are dealt with in a later chapter.

Passive filters, made from combinations of resistors, capacitors and inductors, can **suppress** certain frequencies within a frequency spectrum.

Capacitors

In DC circuits, capacitors usually act as a break in the circuit - they prevent the flow of a continuous current.

In AC circuits, however, their behaviour seems different. A continuous AC current can flow. Capacitors limit this current in a way that resembles the behaviour of resistors in DC circuits. They do not have resistance, as such, but have **capacitive reactance**. This measures the opposition of the capacitor to an AC current.

It is given the symbol $\mathbf{X_C}$ and is measured in ohms (Ω).

The following equation is used to calculate capacitive reactance at a given frequency, f:

$$X_C = \frac{1}{2\pi fC}$$

Here:

- X_C is the reactance, measured in ohms
- f is the frequency of the AC signal measured in hertz
- C is the capacitance in farads.

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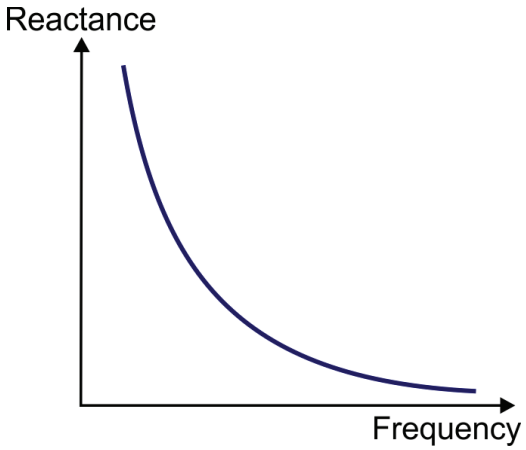
The graph shows how capacitive reactance changes as the frequency of the signal changes.



The capacitor behaves as a frequency-dependent resistor. It has a huge effect on the AC current when the frequency is low but a much smaller effect as the frequency rises.

Impedance

When considering the combined effect of resistors and capacitors in a circuit, we cannot simply add together resistance and reactance. We define a new term, **impedance**, as their combined effect. It is given the symbol **Z** and is measured in ohms.

The formula for calculating impedance is: $Z = \sqrt{R^2 + X_C^2}$

Example:

A 2.2kΩ resistor is connected in series with a 47nF capacitor.

Calculate the reactance of the capacitor and the impedance of the circuit at:

(a) 100 Hz

(b) 10 kHz.

(a) at 100 Hz $X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{2200^2 + 33863^2} = 39634\Omega$

(b) at 10 kHz $X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 10000 \times 47 \times 10^{-9}} = 338.6\Omega$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{2200^2 + 338.6^2} = 2226\Omega$

At low frequencies, the capacitor is the dominant component whereas, at high frequencies, the resistor is the main contributor.

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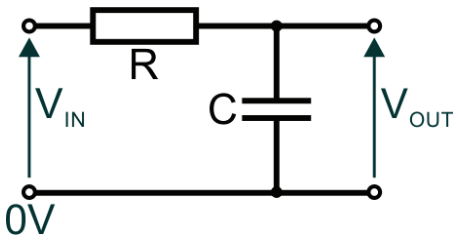
Low-pass filter

A low-pass filter (LPF) is used to remove high frequency components from a signal spectrum.
The circuit consists of a resistor in series with a capacitor. The output voltage is taken across the capacitor as shown:



Considering this circuit as a voltage divider, the output voltage can be calculated using a modified version of the voltage divider formula:

$$V_{OUT} = \frac{V_{IN}}{Z} \times X_C$$

$$= \frac{V_{IN}}{\sqrt{R^2 + X_C^2}} \times X_C$$

This can be re-arranged to calculate the voltage gain of the circuit:

$$\text{Voltage gain} = \frac{V_{OUT}}{V_{IN}} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$

At a very **low** frequency: $X_C^2 >> R^2$

'$\mathbf{R}^2$' will be so small compared to $\mathbf{X}_C^2$ that it can be ignored and the formula reduces to:

$$\text{Voltage gain} = \frac{X_C}{\sqrt{X_C^2}} = 1$$

At a very **high** frequency: $R^2 >> X_C^2$

Now ignoring $\mathbf{X}_C$, the formula reduces to:

$$\text{Voltage gain} = \frac{X_C}{\sqrt{R^2}} = \frac{X_C}{R} = \frac{1}{2\pi f C R}$$

This voltage gain is less than 1, meaning that high frequency signals will be suppressed.

What happens to mid-range frequencies?

As the frequency is increased from a low value, at some point the value of $\mathbf{X}_C$ becomes equal to **R**. This frequency marks the transition between the two extremes and is given a special name -the **break frequency** ($\mathbf{f_b}$).
By definition, then, at the break frequency: $R = X_C = \frac{1}{2\pi f_b C}$ *f_b:*

which can be re-arranged as: $f_b = \frac{1}{2\pi R C}$

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At the break frequency: $X_c = R$

Substituting this into the voltage gain formula:

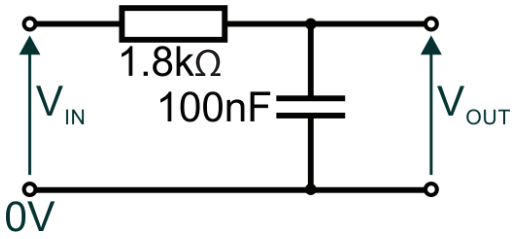
$$\text{Voltage gain} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{X_C}{\sqrt{X_C^2 + X_C^2}} = \frac{1}{\sqrt{2}}$$

In other words: $$V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN} = 0.707 \times V_{IN}$$

At the break frequency, then V_{OUT} will be approximately 0.7 x V_{IN}.

This occurred earlier when dealing with the bandwidth of op-amps.

Example: For the low-pass filter shown in the diagram, V_{IN} = 10 V.



Calculate:

(a) the reactance of the capacitor at 10 Hz, 1 kHz, and 100 kHz

(b) the output voltage at each of these frequencies

(c) the break frequency of this circuit

(d) V_{OUT} at the break frequency.

(a) At 10 Hz: $$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 10 \times 100 \times 10^{-9}} = 159{,}154\Omega$$

At 1 kHz: $$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 1000 \times 100 \times 10^{-9}} = 1{,}591\Omega$$

At 100 kHz: $$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 100 \times 10^{3} \times 100 \times 10^{-9}} = 15.9\Omega$$

(b) At 10 Hz: $$V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{159154}{\sqrt{1800^2 + 159154^2}} \times 10 = 9.99V$$

At 1 kHz: $$V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{1591}{\sqrt{1800^2 + 1591^2}} \times 10 = 6.62V$$

At 100 kHz: $$V_{OUT} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \times V_{IN} = \frac{15.9}{\sqrt{1800^2 + 15.9^2}} \times 10 = 0.09V$$

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(c) Break frequency $f_b = \frac{1}{2 \pi R C} = \frac{1}{2 \times \pi \times 1800 \times 100 \times 10^{-9}} = 884.2\text{Hz}$

(d) At break frequency $V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN} = 0.707 \times 10 = 7.1\text{V}$

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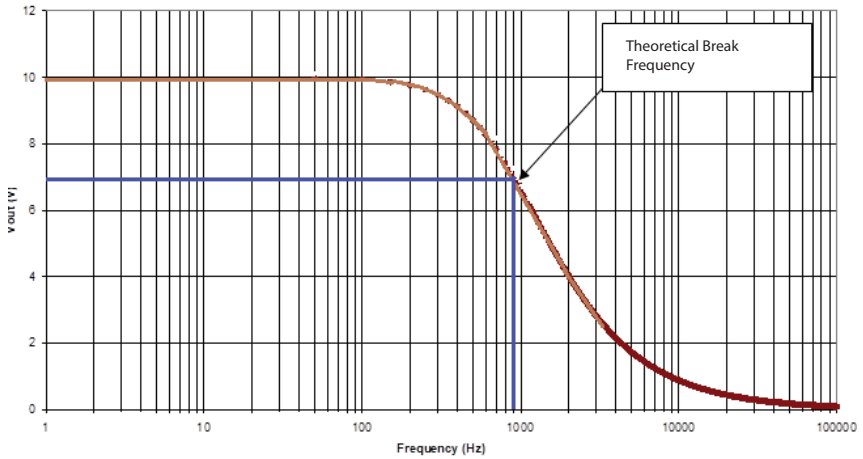
Effect of frequency on voltage gain

Note: since the voltage gain is always less than or equal to unity for passive filters, the frequency response graphs are plotted as V_{OUT} against frequency.

The graph shows how output voltage changes with frequency for the circuit in the example above, (on linear-log graph paper).



A line drawn across at 7.07 V gives a break frequency of approximately 900 Hz.
The shape of this frequency spectrum is not that of an **ideal** low-pass filter. The 'knee' as the trace approaches the break frequency is caused by the behaviour of the capacitor. Its reactance changes continuously with frequency. It doesn't suddenly switch from being 'high' to being 'low'.
Comparing the graph to the ideal low-pass filter characteristic, there are several key differences:

- there is a roll off in gain as the break frequency is approached;
- gain slowly falls over a range of frequencies - no vertical drop at the break frequency;
- there is a small output voltage even at high frequencies.

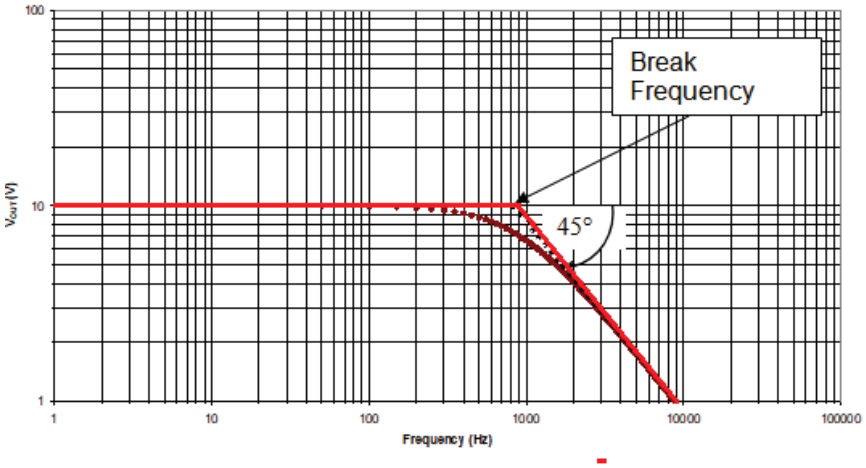
Despite all these deviations from the ideal, in practice this approach works well.

Two line approximation of a frequency response

We can predict the frequency response of a filter by representing the response by two straight lines (on log-log graph paper). To do so for the above filter:

- draw a horizontal line at 10 V as far as the break frequency (900 Hz);
- draw a second line at 45º from the end of the first line to cut the frequency axis.

This two line approximation is shown as the red line in the next graph.



With the two-line approximation, a vertical line through the break frequency passes through the theoretical break frequency on the accurate response (shown in brown).

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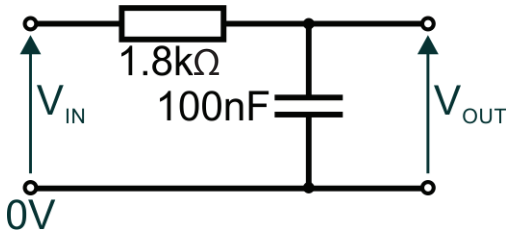
Investigation 4.2

Set up the low-pass filter shown on the right.

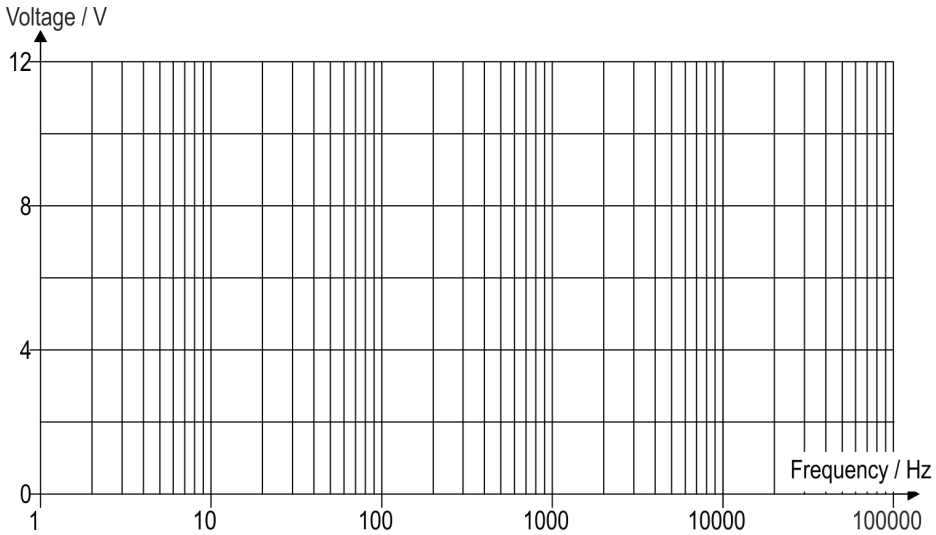


(a) Connect a function generator set to produce a 10 Hz sinewave output of amplitude 10 V. Use an oscilloscope to observe both V_{IN} and $V_{OUT.}$

(b) Complete the table below by recording the values of $\mathbf{V_{OUT}}$ at each frequency.

Frequency / Hz	V_{IN} / V	V_{OUT} / V
10	10	
100	10	
200	10	
400	10	
600	10	
800	10	
1000	10	
2000	10	
4000	10	
6000	10	
8000	10	
10000	10	

(c) Use your results to plot the frequency response on the linear/log graph grid below.



(d) Use the graph to determine the break frequency of the filter. .....

(e) Compare the result with the graph provided at the top of page 15.

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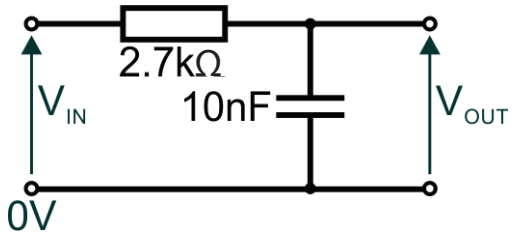
Exercise 4.2

For the low-pass filter shown in the diagram, V_{IN} = 12 V



(a) Calculate:

(i) the reactance of the capacitor at 10 Hz, and 100 kHz
(ii) the output voltage at each of these frequencies
(iii) the break frequency of this circuit
(iv) V_{OUT} at the break frequency.

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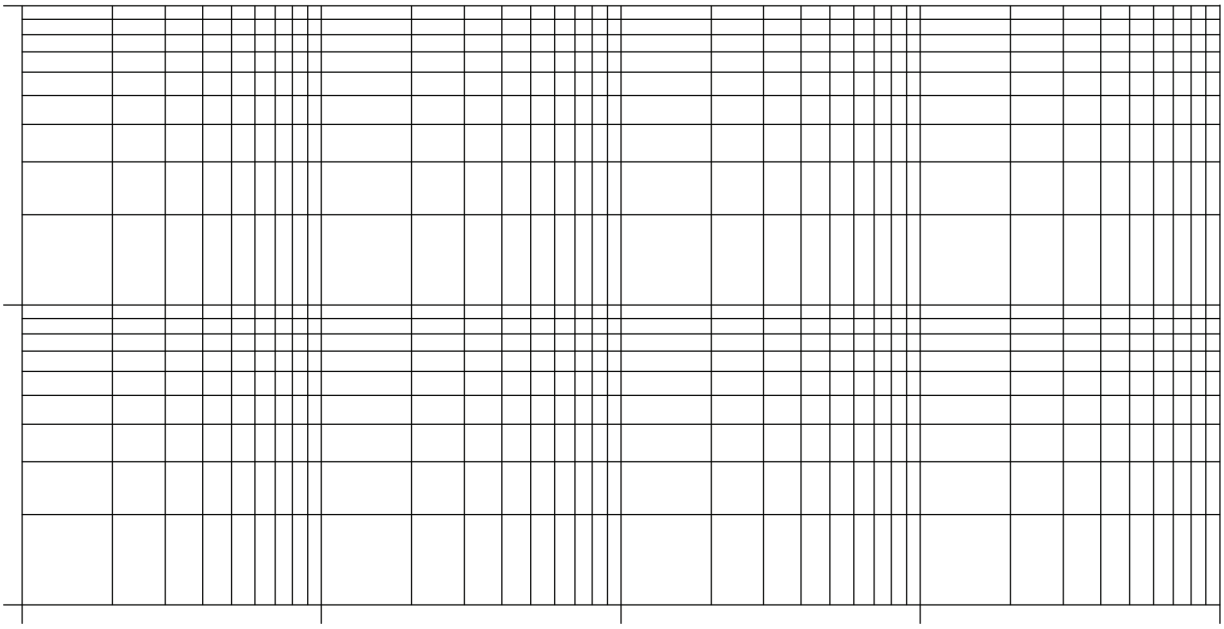
(b) Plot a graph of output voltage against frequency on log-log graph paper, using the two-line approximation.



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Designing a low-pass filter

The previous section looked at how to obtain the frequency response graph for a given circuit. This looks at the reverse - designing a low-pass filter to meet a given frequency response.

Example:

Design a low-pass filter with a break frequency of 2 kHz. The filter uses a 22 nF capacitor.

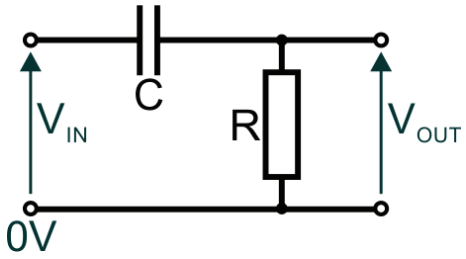
The break frequency can be rearranged to find R as follows:

$$R = \frac{1}{2\,\pi\,f_b\,C} = \frac{1}{2 \times \pi \times 2000 \times 22 \times 10^{-9}} = 3617\Omega$$

A resistor of 3617 Ω, would give the exact breakpoint required, but is not available in the range of preferred values. The nearest preferred values are 3.3 kΩ and 3.9 kΩ. If necessary a variable resistor could be connected in series with a 3.3 kΩ resistor and adjusted to give the exact break frequency.

High-pass filter

A high-pass filter is used to remove the low frequency components from a signal. The circuit is straightforward, with the capacitor and resistor of the low-pass filter circuit interchanged, so that the output signal is the voltage across the resistor.



As before, considering the arrangement as a voltage divider, the output voltage is given by:

$$V_{OUT} = \frac{V_{IN}}{Z} \times R$$

$$= \frac{V_{IN}}{\sqrt{R^2 + X_C^{\,2}}} \times R$$

This formula can be re-arranged to calculate the voltage gain of the circuit:

$$\text{Voltage gain} = \frac{V_{OUT}}{V_{IN}} = \frac{R}{\sqrt{R^2 + X_C^{\,2}}}$$

Looking at the extremes:

- when the frequency is high, $R^2 >> X_C^{\,2}$ and $X_C^{\,2}$ will be so small it can be ignored compared to the size of R^2. The voltage gain will be nearly 1 i.e. no change
- when the frequency is low, $X_C^{\,2} >> R^2$ and so R^2 will be so small it can be ignored compared to the size of $X_C^{\,2}$. The voltage gain will be:

$$\text{Voltage gain} = \frac{V_{OUT}}{V_{IN}} = \frac{R}{X_C} = \frac{R}{1 / 2\,\pi\,f\,C} = 2\,\pi\,f\,C$$

This is smaller than 1 and so the low frequency signals will be suppressed.

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As before, somewhere in between these extremes, the value of R and X_C are equal. The frequency at which this occurs is called the **break frequency** ($\mathbf{f_b}$).

In other words, at the break frequency: $R = X_C = \frac{1}{2 \pi f_b C}$

Re-arranging this gives: $f_b = \frac{1}{2 \pi R C}$

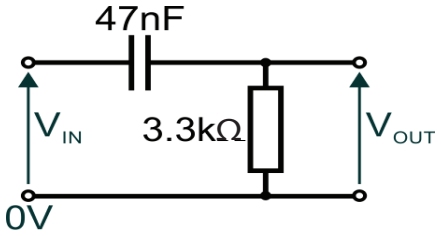
Note: The formula is identical to that for the low-pass filter.

In a similar way the value of V_{OUT} at the break frequency will be given by:

$$V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN} = 0.707 \times V_{IN}$$

Example:

Consider the following circuit:



Calculate:

(a) the reactance of the capacitor at 100 Hz, and 100 kHz

(b) the output voltage at each of these frequencies

(c) the break frequency of this circuit

(d) V_{OUT} at the break frequency when V_{IN} = 10 V.

(a) Calculate the reactance of the capacitor at 100 Hz, and 100 kHz.

At 100 Hz: $X_C = \frac{1}{2 \pi f C} = \frac{1}{2 \times \pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$

At 100 kHz: $X_C = \frac{1}{2 \pi f C} = \frac{1}{2 \times \pi \times 100 \times 10^3 \times 47 \times 10^{-9}} = 33.9\Omega$

or At 100 kHz frequency has increased by a factor of a 1000, so X_C decreases by the same factor:

(b) At 100 Hz: $V_{OUT} = \frac{R}{\sqrt{R^2 + X_C{}^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 33863^2}} \times 10 = 0.97V$

At 100 kHz: $V_{OUT} = \frac{R}{\sqrt{R^2 + X_C{}^2}} \times V_{IN} = \frac{3300}{\sqrt{3300^2 + 33.9^2}} \times 10 = 10.00V$

(c) Break frequency $f_b = \frac{1}{2 \pi R C} = \frac{1}{2 \times \pi \times 3300 \times 47 \times 10^{-9}} = 1026.1Hz$

(d) At the break frequency $V_{OUT} = \frac{1}{\sqrt{2}} \times V_{IN} = 0.707 \times 10 = 7.07V$

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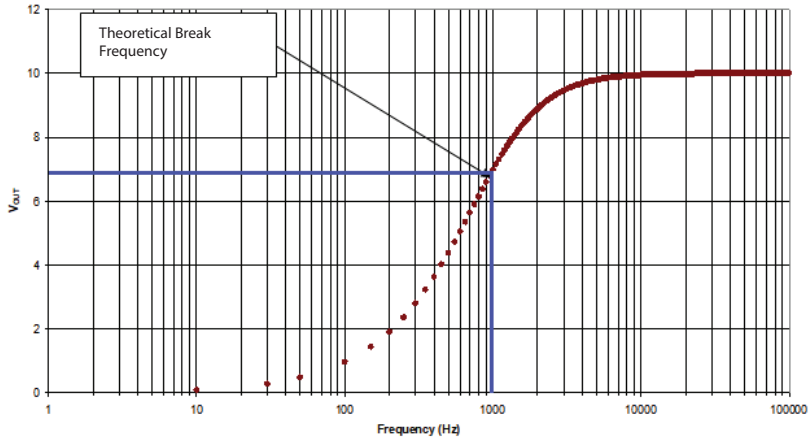
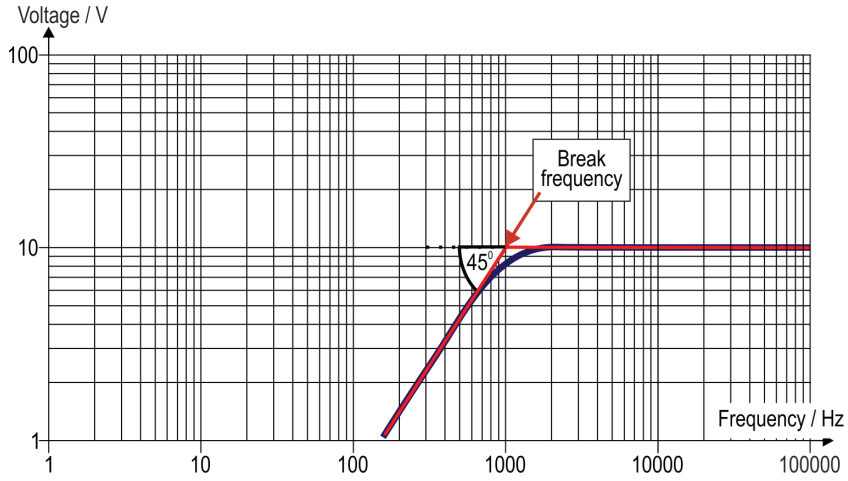
The graph shows how output voltage varies with frequency.



A horizontal line drawn across at 7.07 V obtains a break frequency of approximately 1000 Hz.

The two line approximation, on log-log graph paper, is shown as the thinner red trace.



Buffering the output of a filter

The values R and C in a RC filter determine the frequency response. A problem arises when you connect the filter output to the next stage in a system. For a high-pass filter the input resistance of the next stage will be in parallel with R, changing the effective resistance of the parallel combination of R and the input resistance of the next stage. This will change the frequency response of the filter.

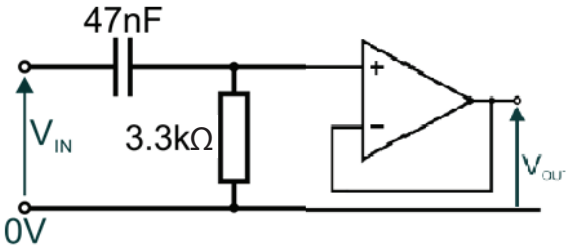
In the example above if the next stage had an input resistance of 367 Ω then the effective value of resistance would change from 3.3 kΩ to 330 Ω (decreased by a factor of 10) causing the break frequency to increase by a factor of 10 to approximately 10,000 Hz.

To avoid this problem, the RC components need to be isolated or buffered from the next stage of a system. This can easily be achieved using a voltage follower circuit as shown below.

The high input resistance of the voltage follower will have no effect on the value R of the filter.

If V_{OUT} needs to be amplified the voltage follower can be replaced with a non-inverting op-amp.

The same principle can be used with low-pass and band-pass filters.



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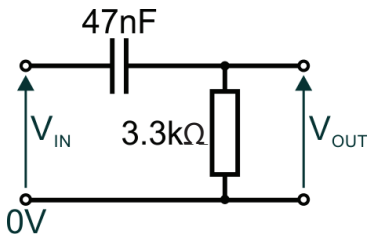
Investigation 4.2

Set up the high-pass filter shown on the right.



(a) Connect a function generator set to produce a 100 Hz sinewave output of amplitude 10V. Use an oscilloscope to observe V_{OUT}.

(b) Complete the third column of the table below by recording the values of $\mathbf{V}_{OUT}$(no load) at each frequency.

Frequency / Hz	**V_{IN} / V**	**V_{OUT} / V (no load)**	**V_{OUT} / V (367 Ω load)**	**V_{OUT} / V (367 Ω buffered load)**
100	10			
1,000	10			
10,000	10			
100,000	10			

Compare the result with the graph provided at the top of page 17.

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(c) Connect a 367 Ω resistor across the filter output. Complete the fourth column of the table by recording the values of $\mathbf{V}_{OUT}$(367 Ω load) at each frequency. Compare the results with those in part (b).

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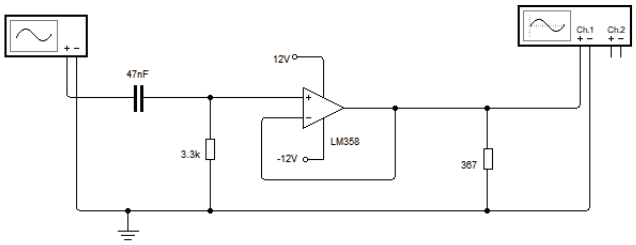
(d) Add an LM358 voltage follower to the circuit as shown on the right.

If you are using Circuit Wizard ensure that the simulation power supply voltage is set to 12 V.



(e) Complete the fifth column of the table by recording the values of $\mathbf{V}_{OUT}$(367Ω buffered load) at each frequency. Compare the results with those in parts (b) and (c) and comment on the effectiveness of the voltage follower as a buffer in this circuit.

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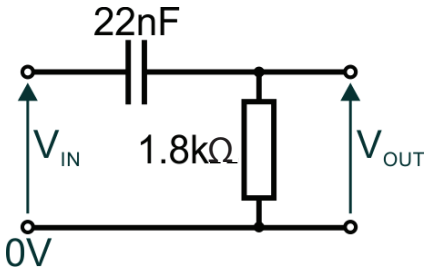
Exercise 4.3

For the high-pass filter shown, V_{IN} = 15 V.



(a) Calculate:

(i) the reactance of the capacitor at 10 Hz, and 100 kHz

(ii) the output voltage at each of these frequencies

(iii) the break frequency of this circuit

(iv) V_{OUT} at the break frequency.

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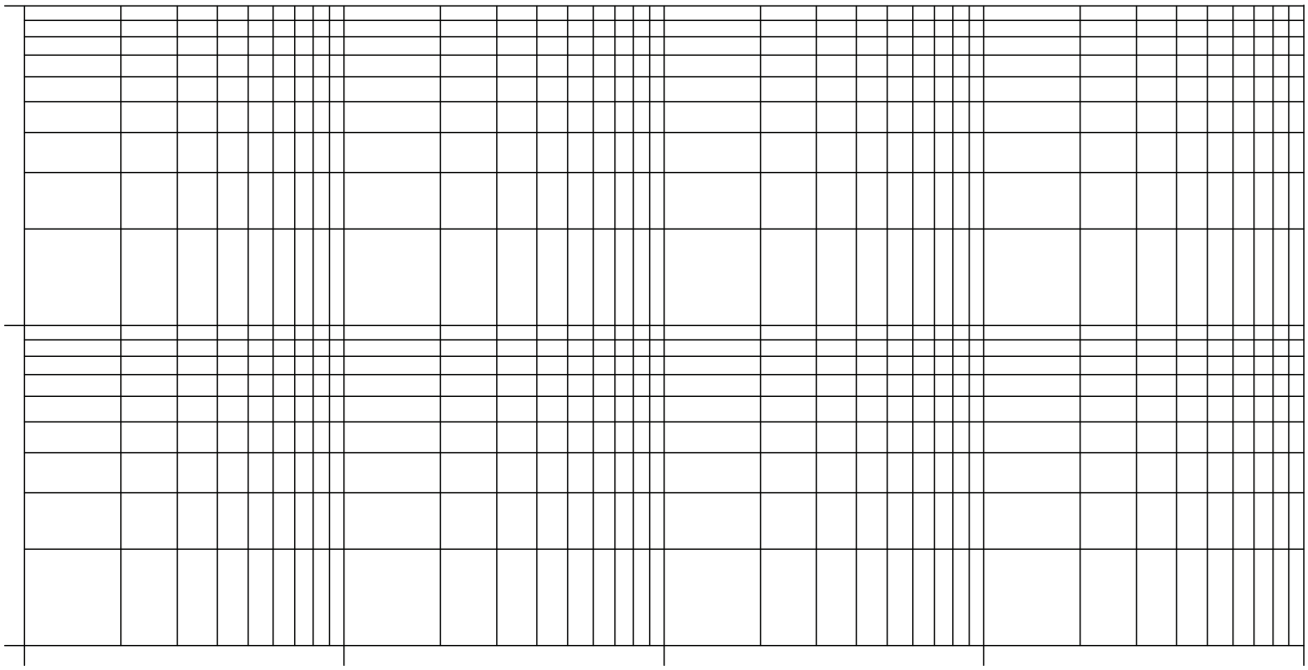
(b) Plot a graph of output voltage against frequency on log-log graph paper, using the two-line approximation.



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Designing a high-pass filter

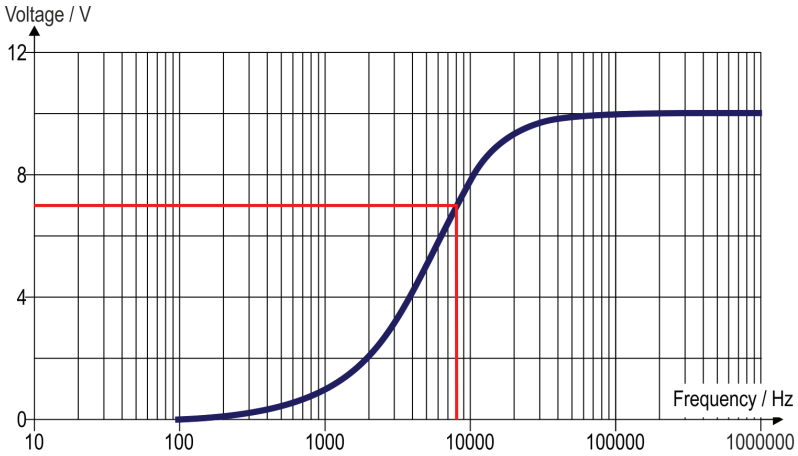
Earlier, we looked at how to obtain the frequency response graph for a given circuit. Now the reverse, designing a high-pass filter to meet a given frequency response.

Example:

Design a high-pass filter to produce the characteristic shown on the right:

The following capacitors are available:
10 μF, 22 nF, and 0.47 pF.

Draw the circuit diagram of the completed filter.



The break frequency is found by drawing a horizontal line across at 0.07 of the maximum voltage until it reaches the characteristic curve.

In this case, the horizontal line, shown in red, is drawn at 7 V and gives a break frequency of 8000 Hz.

(Do not confuse this with the two-line approximation graph plotted on **log-log** graph paper where the break frequency is determined at the point where the graph breaks off at 45°)

Rearranging the break frequency formula: $R = \frac{1}{2 \pi f_b C}$

Calculating the value of R for each available capacitor:

For C=10 μF: $R = \frac{1}{2 \times \pi \times 8000 \times 10 \times 10^{-6}} = 1.99\Omega$

The nearest preferred value is 2Ω.

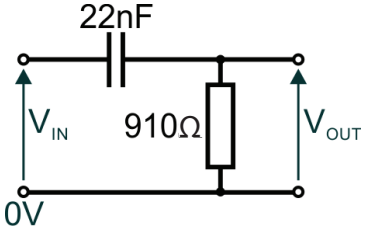
With C=22 nF: $R = \frac{1}{2 \times \pi \times 8000 \times 22 \times 10^{-9}} = 904.29\Omega$

The nearest preferred value is 910Ω.

With C=0.47 pF:: $R = \frac{1}{2 \times \pi \times 8000 \times 0.47 \times 10^{-12}} = 42328442\Omega$

Of these, the 2 Ω resistor is unsuitable. It would draw excessive current from the source. The 42.3 MΩ is beyond the E24 series which stops at 10 MΩ. The choice, then, is 910 Ω.

The completed circuit is:



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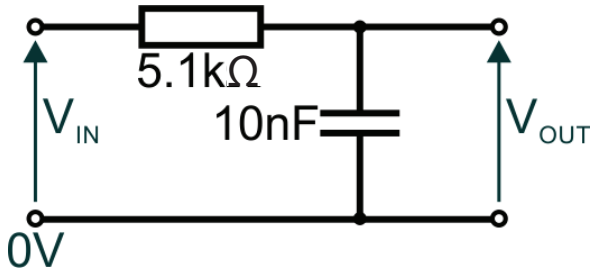
Exercise 4.4:

1. The following circuit shows one form of filter.



(a) What is the name of this type of filter? .....

(b) Calculate the reactance of the capacitor at 100 Hz.

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(c) Estimate the reactance of the capacitor at 10 kHz. .....

(d) Calculate the break frequency for this filter.

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(e) Sketch the characteristic of this filter, labelling all critical values.

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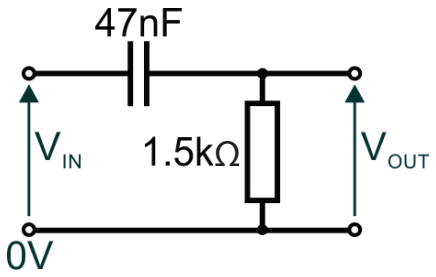
2. The following circuit is used as a filter.



(a) What is the name of this type of filter? .....

(b) Calculate the reactance of the capacitor at 3000 Hz.

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(c) What is the impedance of the circuit at 3000 Hz.

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(d) Calculate the output voltage at 3000 Hz if V_{IN} = 5 V.

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(e) Calculate the break frequency for this filter.

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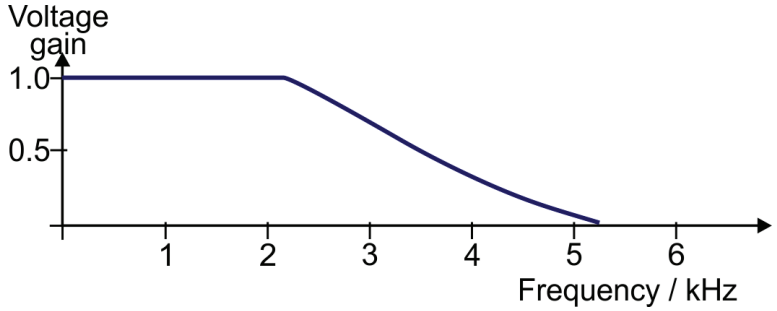
3. Design a filter that has the characteristic shown in the graph.
Assume that all values of capacitor and resistor are available.



(a) What is the break frequency of this filter? .....

(b) Draw the circuit diagram of the filter.

(c) Determine the value of the components required for this filter.

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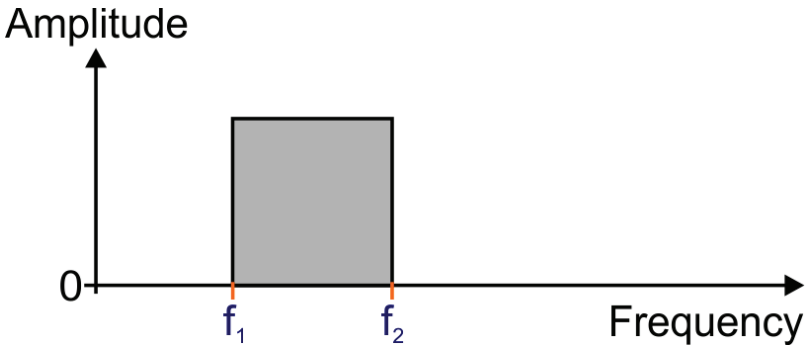
Band-pass Filter

One of the most important circuits used in communication systems is the **band-pass filter (BPF)** or **resonant filter**. These are used usually at frequencies much higher than the audio range.

The ideal band-pass filter has the following characteristic.



It passes only a narrow range of frequencies - greater than f_1 and less than f_2.

Inductors

Resonant filters incorporate an additional component called an **inductor**.

This is a coil of wire, wrapped around a small ferrite rod. Its symbol is shown below.

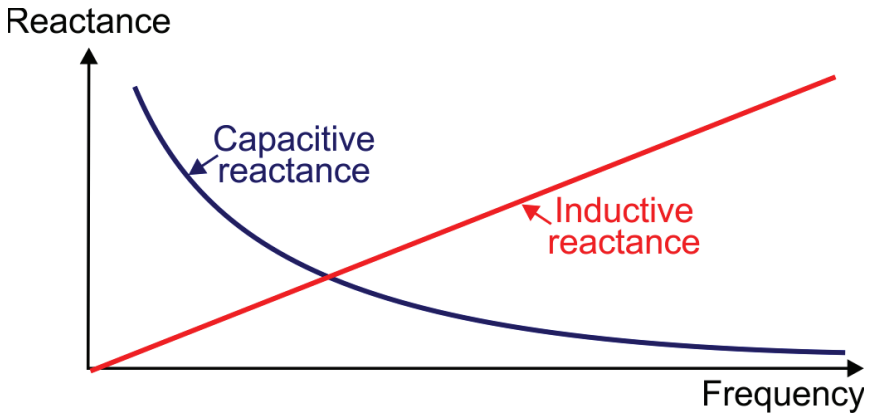
Inductance is measured in Henries (H). Practical inductors are usually measured in millihenries (mH) or microhenries (μH).

In DC circuits, inductors usually act as a wire - they pass a continuous current with little opposition.

In AC circuits, however, this opposition is much bigger. Both inductors and capacitors limit AC in a way that resembles the behaviour of resistors in DC circuits. This opposition to the flow of an AC current is called reactance. Capacitors have **capacitive reactance** ($\mathbf{X}_C$) and inductors have **inductive reactance** ($\mathbf{X}_L$). Both are measured in ohms (Ω).
In many ways, inductors and capacitors behave as mirror images of each other. As the AC frequency increases, inductive reactance increases whereas capacitive reactance decreases.

Earlier, the formula for capacitive reactance was given as: $X_c = \frac{1}{2 \pi f C}$

The equivalent formula for inductive reactance is: $X_L = 2 \pi f L$
The following graph illustrates the behaviour of both capacitors and inductors as the frequency of an AC signal changes.



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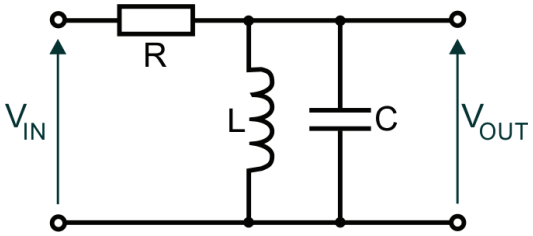
The graph shows that inductive reactance increases linearly with frequency whereas capacitive reactance decreases non-linearly.

The resonant filter circuit consists of a parallel combination of a capacitor, **C**, and an inductor, **L**. A resistor, **R**, is connected in series with this combination to reduce the current drawn from the source at very high and very low frequencies.



To analyse this circuit and calculate $\mathbf{V}_{OUT}$ at different frequencies is complicated and beyond the scope of this course. We can examine what happens under specific circumstances, however. (Refer to the previous graph, while following through these special cases.)

Case 1: The input frequency is very low.

In this case, the reactance of the inductor is very low. The reactance of the capacitor is very high. Most current flows through the inductor rather than the capacitor. The output voltage is negligibly small.

Case 2: The input frequency is very high.

Now, the reactance of the inductor is very high but the reactance of the capacitor is very small. Current now flows mainly through the capacitor as it has a lower reactance. Again, the output voltage is negligibly small.

Case 3: Mid-range frequencies.

Here, the reactance of both inductor and capacitor is significant. Both branches of the parallel circuit have appreciable reactance. At one very special frequency, called the **resonant** frequency, the reactance of the inductor and capacitor are equal and the output voltage will be at a maximum.

Unfortunately the calculation of the effective reactance in parallel is complex and beyond the scope of this course.

However, it is possible to calculate the resonant frequency, $\mathbf{f}_0$, of the circuit, the frequency where inductive reactance $\mathbf{X}_L = \mathbf{X}_C$, the capacitive reactance.

$$X_L = X_C$$

$$2 \pi f_0 L = \frac{1}{2 \pi f_0 C}$$

$$f_0^2 = \frac{1}{4 \pi^2 L C}$$

$$f_0 = \frac{1}{2 \pi \sqrt{LC}}$$

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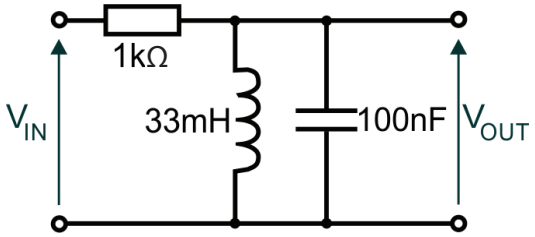
Example: Calculate the resonant frequency of the following band-pass filter.



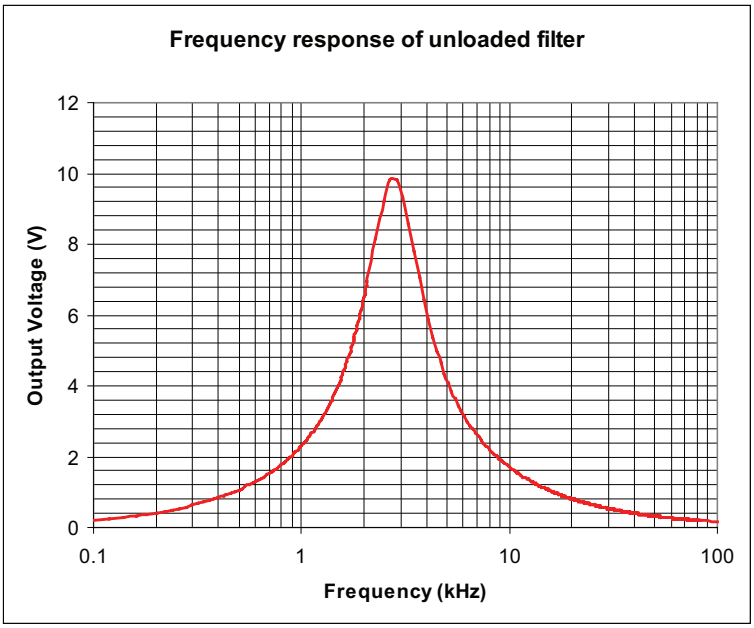
Resonant frequency f_0 is given by:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

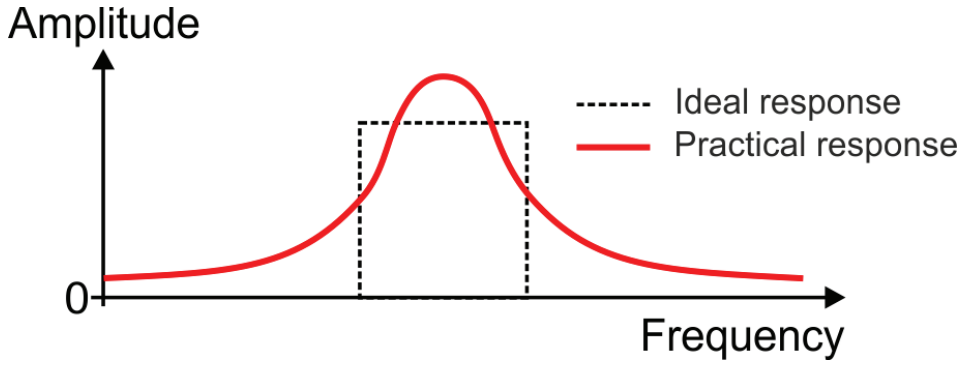
$$= \frac{1}{2\pi\sqrt{33 \times 10^{-3} \times 100 \times 10^{-9}}}$$

$$= 2771\text{Hz}$$

The frequency response of this filter plotted on an Excel spreadsheet is:



This response is not the same as that from an ideal band-pass filter, as the following illustration shows:



There are several key differences between the real and practical responses:

- the voltage gain decreases slowly over a range of frequencies, i.e. no vertical cut-off
- there is a gradual roll-off in voltage gain at both low and high frequencies.

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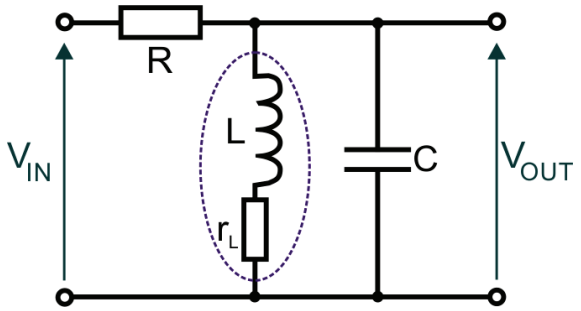
Note:

In obtaining these results, we have assumed ideal properties for all components. In practice, real inductors have a small resistance, $\mathbf{r_L}$, resulting from the wire used to make the coil.
This lowers the resonant frequency slightly.

A more realistic circuit for the band-pass filter is shown below.



The examination specification requires us to calculate the output voltage at one specific frequency only - the resonant frequency. To do this, we calculate the **dynamic resistance** ($\mathbf{R_D}$) of the circuit.

We then replace the parallel combination of inductor and capacitor with a resistance $\mathbf{R_D}$, reducing the circuit to a simple voltage divider consisting of **R** in series with $\mathbf{R_D}$.

This simplification can be applied **only**:

- at resonance
- when the circuit is unloaded, (not delivering an output current).

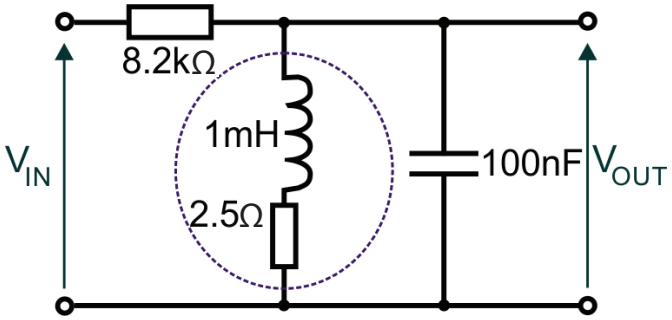
In deriving the formula for $\mathbf{R_D}$ several assumptions are made, in particular that $\mathbf{r_L}$ is small ($\mathbf{r_L} < 25\Omega$).

With this limitation, the formula for the dynamic resistance, $\mathbf{R_D}$, is:

$$R_D = \frac{L}{r_L C}$$

Example:

For the band-pass filter shown in the diagram:



Calculate:

(a) the resonant frequency
(b) the output voltage at resonance.

(a) $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{1 \times 10^{-3} \times 100 \times 10^{-9}}}$$

$$= 15915\text{Hz}$$

(b) $$R_D = \frac{L}{r_L C} = \frac{1 \times 10^{-3}}{2.5 \times 100 \times 10^{-9}} = 4000\Omega$$

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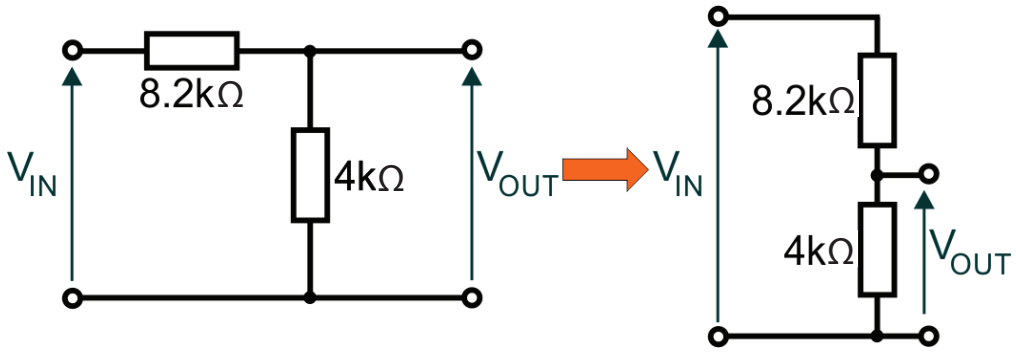
At resonance, the circuit simplifies to a voltage divider:



The circuit diagram on the right shows this simplification drawn in the more usual format for a voltage divider.

Using the voltage divider formula when V_{IN} = 10 V:

$$V_{OUT} = V_{IN} \times \frac{R_D}{R_D + R} = 10 \times \frac{4}{4 + 8.2} = 3.28V$$

showing that, at the resonant frequency, the output voltage is high - as expected.

Summary

- a resonant filter consists of an inductor and capacitor in parallel
- the reactance of an inductor is given by $X_L = 2 \pi f L$
- at resonance, $X_L = X_C$ and the resonant frequency is given by $f_0 = \frac{1}{2 \pi \sqrt{LC}}$
- in practical circuits, the inductor has a small resistance r_L
- at resonance (**only**), the output voltage can be calculated by using the dynamic resistance R_D of the circuit, given by $R_D = \frac{L}{r_L C}$

Sometimes, we need to calculate the value of C or L from the resonant frequency formula.

This can be arranged to give: $C = \frac{1}{4 \pi^2 f_0^2 L}$ and $L = \frac{1}{4 \pi^2 f_0^2 C}$

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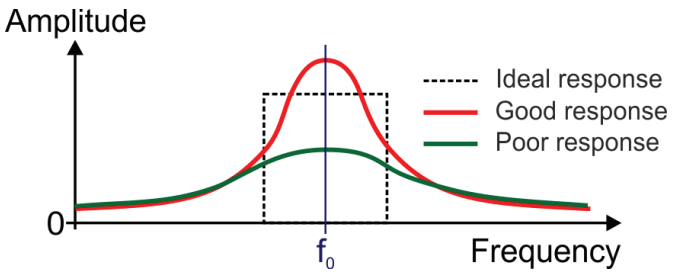
Q-factor and selectivity

The response from the filter, just considered, does not match the ideal characteristic for a band-pass filter.

The **R** and **C** values chosen affect the shape of the frequency response of the filter quite dramatically.



- The red trace shows a focused, though not ideal, response, with a sharp peak at the resonant frequency f_0. On either side of it, the response falls away rapidly.
- The green trace has a flatter response with a less well defined peak, though still at f_0.

The correct term for this feature is **selectivity** or **Q-factor** for the filter.

The Q-factor can be determined in one of three ways:

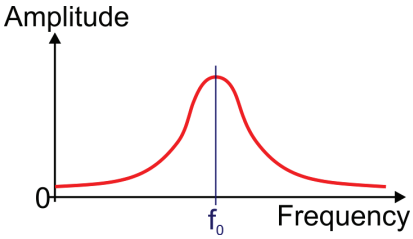
1. using the formula $Q = \frac{2\pi f_0 L}{r_L}$

   where r_L is the resistance of the inductor;

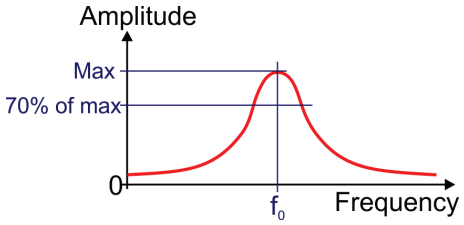
2. using the formula $Q = \frac{f_0}{\text{bandwidth}}$

3. from the graph of the frequency response of the filter:

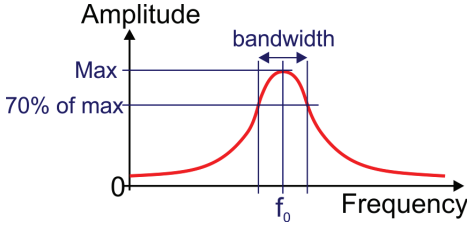
   Step 1. Find f_0, at the peak of the response curve.



   Step 2. Find the amplitude of the peak of the response curve and locate the frequency at which the amplitude is 70% of this value.



   Step 3. Find bandwidth from frequency response.



   Step 4. Use the formula $Q = \frac{f_0}{\text{bandwidth}}$ to calculate Q

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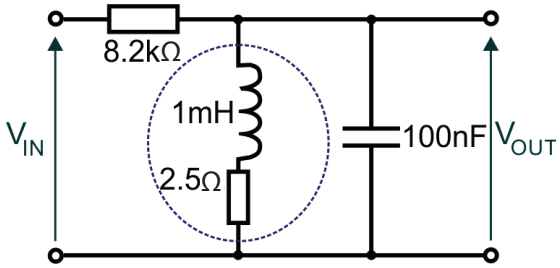
Note:

- the Q-factor of a circuit is dimensionless and has no units, just like voltage gain
- a good filter will have a high Q-factor, but not too high. Too high a Q-factor results in a very narrow bandwidth, possibly cutting out some important signal frequency components
- the design of a band-pass filter is not an easy task. It is relatively easy to obtain a high Q-factor, or to obtain a large bandwidth. It is not easy to achieve both at the same time.
  The design usually involves a compromise
- Adding an external resistor, R, in series with the parallel LC combination in a parallel resonant circuit, will decrease the overall value of Q and increase bandwidth.
  To minimize this effect on Q, it is assumed that the value of the resistor R >> $2\pi f_0 L$.

We now calculate the Q-factor and bandwidth for the band-pass filter circuit explored earlier.

The resonant frequency was calculated earlier as 15915 Hz.



The Q-factor of the circuit is given by:

$$Q = \frac{2\pi f_0 L}{r_L} = \frac{2 \times \pi \times 15915 \times 1 \times 10^{-3}}{2.5} = 40$$

Using these values for f_0 and Q-factor, we can calculate the bandwidth of the filter:

$$Q = \frac{f_0}{\text{bandwidth}} \rightarrow \text{Bandwidth} = \frac{f_0}{Q}$$

$$\text{Bandwidth} = \frac{f_0}{Q} = \frac{15915}{40} = 397.9\text{Hz}$$

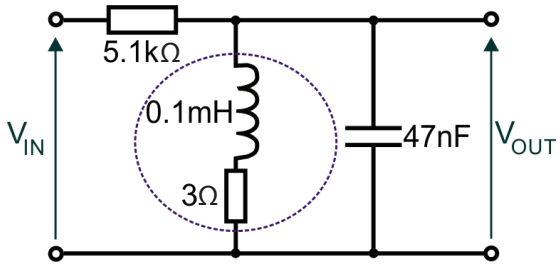
This example demonstrates that it is quite easy to achieve a high Q-factor. However, the impact of this high Q-factor on the bandwidth is severe, resulting in a bandwidth too small for conventional speech signals.

Examples:

1. The following circuit diagram shows a band-pass filter connected to a signal generator, with V_{IN} set to 10 V.

   The inductor has a resistance r_L of 3 Ω.

   V_{IN} is kept at 10 V as the frequency is increased to find the maximum value of V_{OUT}.



(a) Calculate the frequency at which V_{OUT} is a maximum.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 10^{-3} \times 47 \times 10^{-9}}} = 73413\text{Hz} = 73.4\text{kHz}$$

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(b) Calculate the dynamic resistance, R_D, of the filter and hence determine the maximum value of the voltage V_{OUT} when V_{IN} is set to 10 V.

$$R_D = \frac{L}{r_L C} = \frac{0.1 \times 10^{-3}}{3 \times 47 \times 10^{-9}} = 709.2\Omega$$

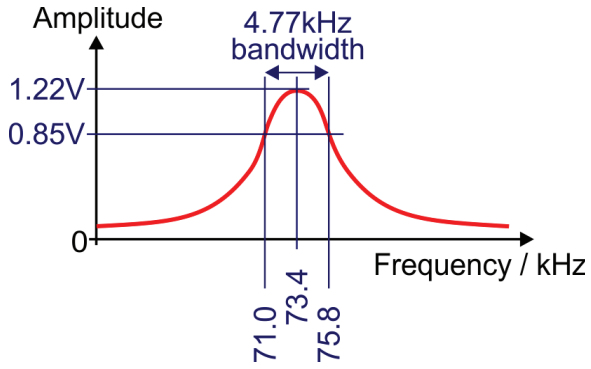
$$V_{OUT} = V_{IN} \times \frac{R_D}{R_D + R} = \frac{10 \times 709}{709 + 5100} = 1.22V$$

(c) Determine the bandwidth of this filter.

$$Q = \frac{2\pi f_0 L}{r_L} = \frac{2 \times \pi \times 73413 \times 0.1 \times 10^{-3}}{3} = 15.38$$

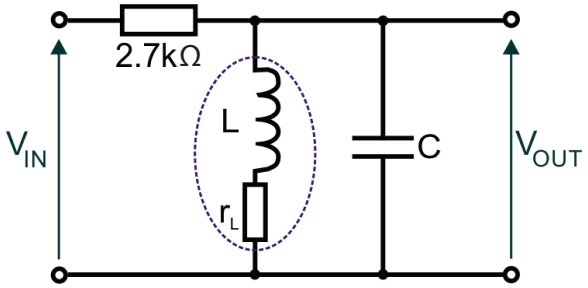
$$\text{Bandwidth} = \frac{f_0}{Q} = \frac{73413}{15.38} = 4773Hz$$

(d) Use the axes below to sketch the frequency response of the filter.
Label all important values.



2. The following circuit diagram shows a band-pass filter connected to a signal generator with V_{IN} set to 12 V.

V_{IN} is kept at 12 V while the frequency is increased to find the maximum value of V_{OUT}.



The filter has a Q-Factor of 10, and a bandwidth of 10 kHz.

(a) Calculate the resonant frequency of the filter.

$$Q = \frac{f_0}{\text{bandwidth}} \rightarrow f_0 = Q \times \text{Bandwidth} = 10 \times 10 = 100kHz$$

(b) Three inductors, **X**, **Y** and **Z**, are available.

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They have the following properties:

Inductor **X** L=50 mH, r_L = 2.45 Ω

Inductor **Y** L=50 μH, r_L = 3.15 Ω

Inductor **Z** L=50 μH, r_L = 2.45 Ω.

Perform calculations to enable you to decide which inductor is the correct one to use in this circuit.

Inductor **X** : $Q = \frac{2\pi f_0 L}{r_L} = \frac{2 \times \pi \times 100 \times 10^3 \times 50 \times 10^{-3}}{2.45} = 12823$

Inductor **Y** : $Q = \frac{2\pi f_0 L}{r_L} = \frac{2 \times \pi \times 100 \times 10^3 \times 50 \times 10^{-6}}{3.15} = 9.97$

Inductor **Z** : $Q = \frac{2\pi f_0 L}{r_L} = \frac{2 \times \pi \times 100 \times 10^3 \times 50 \times 10^{-6}}{2.45} = 12.82$

The correct inductor is **Y**, as this gives a Q-factor closest to the specification (10).

(c) Using the inductor chosen in (b), calculate the corresponding value of the capacitor **C** that meets the specification.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \rightarrow f_0^2 = \frac{1}{4\pi^2 LC} \rightarrow C = \frac{1}{4\pi^2 L f_0^2}$$

$$= \frac{1}{4 \times \pi^2 \times 50 \times 10^{-6} \times (100 \times 10^3)^2}$$

$$= 50.66\text{nF} \rightarrow 51\text{nF}$$

(d) Hence calculate the value of R_D at resonance.

$$R_D = \frac{L}{r_L C} = \frac{50 \times 10^{-6}}{3.15 \times 51 \times 10^{-9}} = 313\Omega$$

(e) Hence calculate the output voltage at resonance.

$$V_{OUT} = V_{IN} \times \frac{R_D}{R_D + R} = 12 \times \frac{313}{313 + 2700} = 1.25\text{V}$$

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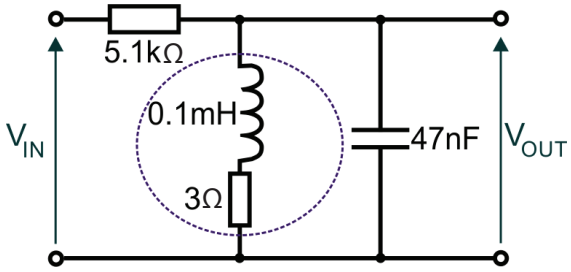
Investigation 4.3

Set up the band-pass filter shown on the right.

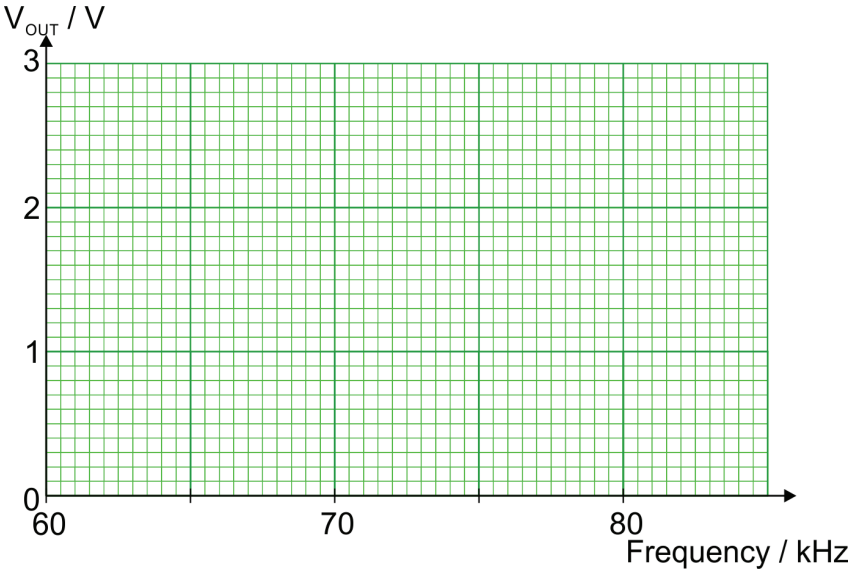


(a) Connect a function generator set to produce a 68 kHz sine wave output of amplitude 10 V. Use an oscilloscope to observe V_{OUT}. Record the result in the table below.

(b) Complete the table by recording the values of $\mathbf{V_{OUT}}$ at each of the other frequencies

Frequency / kHz	V_{IN} / mV	V_{OUT} / V
68	10	
69	10	
70	10	
71	10	
72	10	
73	10	
74	10	
75	10	
76	10	
77	10	
78	10	
79	10	
80	10	

(c) Use your results to plot the frequency response on the axes below.



(d) Determine the resonant frequency and bandwidth of the filter and compare the values with those provided in example 1 on page 36.

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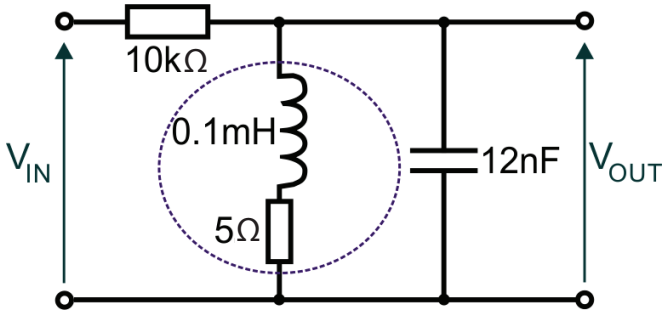
Exercise 4.5

1. The circuit diagram for a band-pass filter is shown below.
The filter is connected to a signal generator which provides an input voltage, V_{IN}, of 10 V.
The inductor has a resistance, r_L, of 5 Ω.



Keeping the input voltage constant at 10 V, the signal frequency is increased.

(a) Calculate the frequency at which V_{OUT} is a maximum.

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(b) By calculating the dynamic resistance, R_D, of the filter, determine the maximum value of the voltage V_{OUT}.

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(c) Determine the bandwidth of the filter.

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(d) Using the axes below, sketch the frequency response of the filter.
Label all important values.

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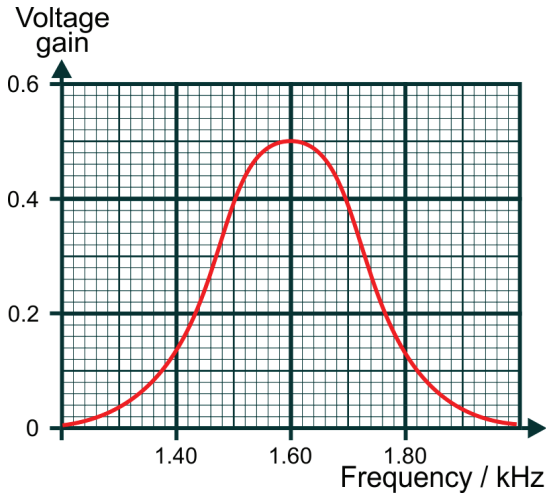
2. The following graph was plotted by a student investigating the behaviour of a band-pass filter.



(a) Determine the resonant frequency of the filter. .....

(b) Determine the maximum gain that can be obtained from this filter. .....

(c) Determine the bandwidth of the filter.

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(d) Hence calculate the Q-Factor for the filter.

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(e) The student connected the filter to a load and observed that the behaviour of the filter changed dramatically. Draw the circuit diagram for the filter including any components needed to overcome this problem.

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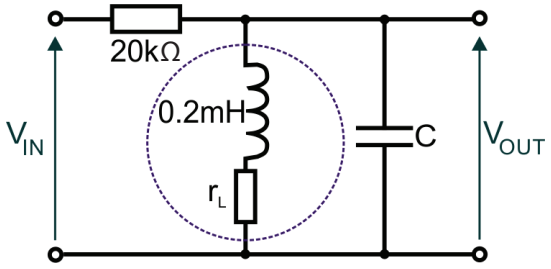
3. The circuit diagram for another band-pass filter is shown in the diagram opposite.
The filter is connected to a signal generator which provides an input voltage, V_{IN}, of 12 V.
The filter is required to have a Q-factor of 40, and a bandwidth of 2.4 kHz.



(a) Calculate the resonant frequency of the filter.

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(b) Use your answer to (a) to calculate the value of the capacitor needed to achieve this resonant frequency.

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(c) Use your answers and the information given above to calculate the value of r_L, the resistance of the inductor.

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(d) Hence calculate the value of R_D at resonance.

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(e) Hence calculate the output voltage at resonance.

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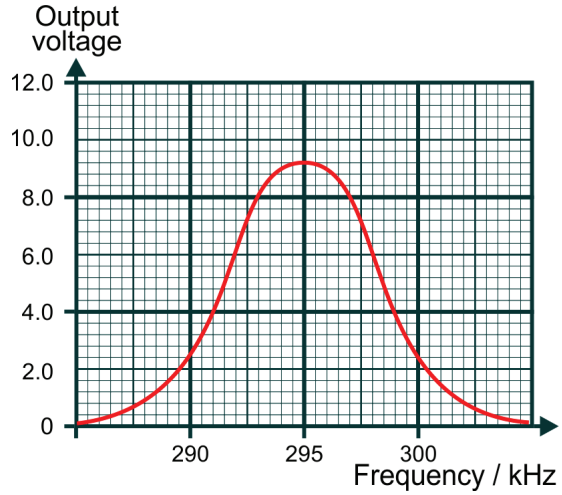
4. The graph shows the frequency response of a band-pass filter.



(a) Determine the resonant frequency of the filter.

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(b) Determine the bandwidth of the filter.

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(c) Hence calculate the Q-Factor for the filter.

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(d) The filter uses a 4.7 nF capacitor. Calculate the value of inductor required to produce the resonant frequency determined in part (a).

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(e) The filter is used as part of a communication system. The input to the next sub-system requires an input with a peak value of approximately 10 V.
Draw a design for a suitable buffer circuit that will produce the required output.
Your diagram should show appropriate component values.

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